Summability methods of statistically convergent sequences

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Definition

Let $x = \{x_k\}_{k=0}^{\infty}$ be a complex sequence. Define

$$\|x\|_p := \left(\sum_{k=0}^{\infty} |x_k|^p\right)^{1/p} \quad (1 \le p < \infty) \text{ and } \quad \|x\|_\infty := \sup_{k \ge 0} |x_k|.$$

We also define

- m: the set of all bounded sequences
- c: the set of all convergent sequences
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(a) $\ell^p \subsetneqq c_0 \subsetneqq c \subsetneqq m$

(b) $(X, \|\cdot\|_\infty)$, *where* $X \in \{m, c, c_0\}$, *and* $(\ell^p, \|\cdot\|_p)$ $(1 \le p < \infty)$ *are both Banach spaces.*

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Let $B = (b_{nk})_{n,k\geq 0}$ be an infinite matrix, and X, Y be two sequence spaces.

(a) For each sequence $x = \{x_k\}_{k=0}^{\infty}$, we set $Bx = \{(Bx)_n\}_{n=0}^{\infty}$ be the sequence defined by

$$(Bx)_n := \sum_{k=0}^{\infty} b_{nk} x_k.$$

(b) The set (X, Y) denotes the collection of all B such that for each $x \in X$, Bx is well-defined and $Bx \in Y$.

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Let $(X, \|\cdot\|)$ and $(Y, \|\cdot\|_*)$ be two semi-normed sequence spaces. Suppose $B \in (X, Y)$.

(a) The semi-norm of the matrix map $B : (X, \|\cdot\|) \to (Y, \|\cdot\|_*)$ given by $x \mapsto Bx$ is defined by

$$\|B\|_{X,Y} = \inf\{M > 0 : \|Bx\|_* \leq M\|x\| \text{ for all } x \in X\}.$$

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Theorem (Silverman-Toeplitz Theorem)

$B = (b_{nk})_{n,k\geq 0}$ is regular if and only if B satisfies

(a) $\sup_{n\geq 0} \sum_{k=0}^{\infty} |b_{nk}| < \infty$,

(b) $\lim_{n\to\infty} b_{nk} = 0$ *for each* $k = 0, 1, \cdots$, *and*

(c) $\lim_{n\to\infty} \sum_{k=0}^{\infty} b_{nk} = 1$.

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A sequence $x = \{x_k\}_{k=0}^{\infty}$ is called statistically convergent to ℓ if for each $\epsilon > 0$,

$$\lim_{n\to\infty} \frac{1}{n+1} |\{k \leq n : |x_k - \ell| \geq \epsilon\}| = 0,$$

where $|K|$ denotes the cardinality of $K \subset \mathbb{N}^0$ (nonnegative integers). In this case, we write $st\text{-}\lim_{k\to\infty} x_k = \ell$ or $x_k \xrightarrow{st} \ell$.

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$$x_k = \begin{cases} n^2 & \text{if } k = n^2 \text{ for some } n \\ 0, & \text{otherwise.} \end{cases} \implies st\text{-}\lim_{k\to\infty} x_k = 0.$$

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$c_0 \underset{\neq}{\subsetneq} st^0 \underset{\neq}{\subsetneq} st$ and $c \underset{\neq}{\subsetneq} st$

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Let $A = (a_{nk})_{n,k \geq 0}$ be a nonnegative regular matrix.

(a) The A-density of a subset K of $\mathbb{N}^0$ is defined by $\delta_A(K) = \lim_{n\to\infty} \sum_{k \in K} a_{nk}$.

(b) A sequence $x = \{x_k\}_{k=0}^{\infty}$ is called A-statistically convergent to ℓ if for each $\epsilon > 0$,

$$\delta_A\Big(\{k : |x_k - \ell| \geq \epsilon\}\Big) = \lim_{n\to\infty} \sum_{k:|x_k-\ell|\geq\epsilon} a_{nk} = 0. \tag{1}$$

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If $A = C_1 = (c_{nk}^{(1)})_{n,k\geq 0}$ is the Cesáro matrix defined by $c_{nk}^{(1)} = \begin{cases} \frac{1}{n+1} & \text{if } k \leq n, \\ 0 & \text{if } k > n, \end{cases}$

$\Longrightarrow$ (1) becomes $\lim_{n\to\infty} \dfrac{1}{n+1}|\{k \leq n : |x_k - \ell| \geq \epsilon\}| = 0.$

$\Longrightarrow$ The C_1-statistical convergence is the original statistical convergence.

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Let $x = \{x_k\}_{k=0}^{\infty}$ be a sequence. x is called A-statistically bounded if there exists a positive number M such that

$$\delta_A(\{k : |x_k| > M\}) = 0. \tag{2}$$

For $A = C_1$, (2) takes the form $\lim_{n\to\infty} \frac{1}{n+1}|\{k \le n : |x_k| > M\}| = 0$. In this case, we say that x is statistically bounded.

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Question: Suppose that A and D are both nonnegative regular matrices.
For which matrix B, we have $B \in (st_A \cap m, st_D)$ holds?

Theorem

Let $B = (b_{nk})_{n,k\geq 0}$. *If* $\{b_{nk}\}_{k=0}^{\infty} \in \ell^1$ *for all* $n = 0, 1, \cdots$, $\left\{ \sum_{k=0}^{\infty} |b_{nk}| \right\}_{n=0}^{\infty}$ *is D-statistically bounded,* $\bar{b}_k = st_D\text{-}\lim_{n\to\infty} b_{nk}$ *exists for each* $k = 0, 1, \cdots$, $\bar{b} = st_D\text{-}\lim_{n\to\infty} \sum_{k=0}^{\infty} b_{nk}$, *and*

$$st_D\text{-}\lim_{n\to\infty} \sum_{k\in K} |b_{nk} - \bar{b}_k| = 0 \text{ for } K \text{ with } \delta_A(K) = 0,$$

then $B \in (st_A \cap m, st_D)$. *Moreover, for* $x = \{x_k\}_{k=0}^{\infty} \in st_A \cap m$,

$$st_D\text{-}\lim_{n\to\infty} (Bx)_n = \left(\bar{b} - \sum_{k=0}^{\infty} \bar{b}_k \right) st_A\text{-}\lim_{k\to\infty} x_k + \sum_{k=0}^{\infty} \bar{b}_k x_k,$$

and $B : (st_A \cap m, \|\cdot\|_\infty) \to (st_D, \|\cdot\|_{st_D})$ *given by* $x \mapsto Bx$ *satisfies*

$$\|B\|_{st_A\cap m, st_D} = \left| \bar{b} - \sum_{k=0}^{\infty} \bar{b}_k \right| + \sum_{k=0}^{\infty} |\bar{b}_k|.$$

Theorem

Let $B=(b_{nk})_{n,k\geq 0}$. If $\{b_{nk}\}_{k=0}^{\infty}\in\ell^1$ for all $n=0,1,\cdots$, $\left\{\sum_{k=0}^{\infty}|b_{nk}|\right\}_{n=0}^{\infty}$ is D-statistically bounded, $\overline{b}_k=st_D\text{-}\lim_{n\to\infty}b_{nk}$ exists for each $k=0,1,\cdots$, $\overline{b}=st_D\text{-}\lim_{n\to\infty}\sum_{k=0}^{\infty}b_{nk}$, and

$$st_D\text{-}\lim_{n\to\infty}\sum_{k\in K}|b_{nk}-\overline{b}_k|=0 \text{ for } K \text{ with } \delta_A(K)=0,$$

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and $B:(st_A\cap m,\|\cdot\|_\infty)\to(st_D,\|\cdot\|_{st_D})$ given by $x\mapsto Bx$ satisfies

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Corollary

Let $B = (b_{nk})_{n,k\geq 0}$. *Then* $B \in (st_A \cap m, c)$ *and*

$$\lim_{n\to\infty} (Bx)_n = st_A\text{-}\lim_{k\to\infty} x_k.$$

for all $x = \{x_k\}_{k=0}^{\infty} \in st_A \cap m$ *if and only if*

$$\sup_{n\geq 0} \sum_{k=0}^{\infty} |b_{nk}| < \infty, \quad \lim_{n\to\infty} \sum_{k=0}^{\infty} b_{nk} = 1$$

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Definition

Let $\alpha \in \mathbb{R}$ with $-\alpha \notin \mathbb{N}$. The Cesàro matrix $C_\alpha = (c_{nk}^{(\alpha)})_{n,k\geq 0}$ of order α is defined by

$$c_{nk}^{(\alpha)} = \begin{cases} \dfrac{\binom{n-k+\alpha-1}{n-k}}{\binom{n+\alpha}{n}} & \text{if } k \leq n, \\ 0 & \text{if } k > n \end{cases}$$

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(a) $C_\alpha \in (st \cap m, c)$ *if and only if* $\alpha > 0$. *Moreover, for all* $x = \{x_k\}_{k=0}^{\infty} \in st \cap m$,

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Corollary

If $\alpha \geq 1$, we have $\Gamma_\alpha \in (st \cap m, c)$ and $\lim_{n\to\infty} (\Gamma_\alpha x)_n = st\text{-}\lim_{k\to\infty} x_k$ for all $x = \{x_k\}_{k=0}^{\infty} \in st \cap m$.

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Let $q = \{q_k\}_{k=0}^{\infty}$ be a nonnegative sequence with $q_0 > 0$. For $n = 0, 1, \cdots$, set $Q_n = \sum_{k=0}^{n} q_k$. The weighted mean matrix $W_q = (w_{nk})_{n,k\geq 0}$ and the Nörlund matrix $N_q = (u_{nk})_{n,k\geq 0}$ are defined by

$$w_{nk} = \begin{cases} \frac{q_k}{Q_n} & \text{if } k \leq n, \\ 0 & \text{if } k > n, \end{cases} \quad \text{and} \quad u_{nk} = \begin{cases} \frac{q_{n-k}}{Q_n} & \text{if } k \leq n, \\ 0 & \text{if } k > n. \end{cases}$$

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Corollary

If $q = \{q_k\}_{k=0}^{\infty}$ is a bounded nonnegative sequence with $q_0 > 0$ and $Q_n \geq cn$ for some $c > 0$ and for all $n = 0, 1, \cdots$, then $W_q, N_q \in (st \cap m, c)$ and

$$\lim_{n\to\infty} (W_q x)_n = \lim_{n\to\infty} (N_q x)_n = st\text{-}\lim_{k\to\infty} x_k$$

for all $x = \{x_k\}_{k=0}^{\infty} \in st \cap m$.

Thank you for your attention!