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Optimal design of viscoelastic dampers using eigenvalue assignment

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SUMMARY

In this study a procedure for determining the optimum size and location of viscoelastic dampers is proposed using the eigenvalue assignment technique. Natural frequencies and modal damping ratios, required to realize a given target response, are determined first by the convex model. Then the desired dynamic structural properties are realized by optimally distributing the damping and stiffness coefficients of viscoelastic dampers using non-linear programming based on the gradient of eigenvalues. This optimization method provides information on the optimal location as well as the magnitude of the damper parameters. The proposed procedure is applied to the retrofit of a 10-story shear frame, and to a three-dimensional structure with an asymmetric plan. The analysis results confirm that the responses of model structures retrofitted by the proposed method correspond well with the given target response.

KEY WORDS: passive control; viscoelastic dampers; optimal design; eigenvalue assignment

INTRODUCTION

Viscoelastic dampers (VEDs) have been acknowledged as efficient energy dissipators for building structures against dynamic loads such as earthquakes and wind loads. Many researchers have investigated methods for the optimum design of supplemental dampers. For example, Zhang and Soong [1] and Shukla and Datta [2] have used a sequential optimization procedure to determine the optimal location of viscoleastic dampers. Gluck *et al.* [3] obtained the optimal damping matrix to obtain story-wise optimum damping distribution using the optimal solution for the linear quadratic regulator problem. Takewaki *et al.* [4] and Singh and Moreschi [5] used a gradient-based approach to obtain optimum distribution of damping devices. Recently,

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Singh and Moreschi [6] used a genetic algorithm to determine optimal size and location of viscous and viscoelastic dampers.

In this study a procedure for determining optimum size and location of viscoelastic dampers is proposed using the eigenvalue assignment technique. Natural frequencies and modal damping ratios of structures required to satisfy a given target response are determined first by the convex model. Then those desired structural parameters are realized by optimally distributing the damping and stiffness coefficients of viscoelastic dampers using non-linear programming based on the gradient of eigenvalues. This procedure of optimization is opposite to the sequential placement of dampers with prescribed properties.

This study was motivated by the observation that the installation of viscoelastic dampers with inherent stiffness and viscosity changes both the natural frequencies and modal damping ratios of a structure. The possible combinations for natural frequency and modal damping ratio which can achieve the desired structural responses are identified first. Then a proper combination is determined considering various design parameters such as dynamic characteristics of the model structures. The next step is to determine the optimum amount and location of dampers using non-linear programming based on the gradient of natural frequency and damping ratio with respect to the damper design parameters. Larger damper parameters are assigned to the place with the larger gradient, and in this way the desired natural frequencies and damping ratios are realized with the least amount of dampers.

The advantage of the proposed procedure is twofold. (1) The optimization method distributes the dampers in accordance with the gradient of eigenvalues, and provides information on the optimal location as well as the magnitude of the damper parameters. (2) A more efficient optimum design process is possible because eigenvalues are assigned taking into account the dynamic characteristics of the structure itself.

The proposed procedure is applied to the retrofit of a 10-story shear frame, and to a three-dimensional structure with an asymmetric plan, to verify the applicability of the proposed method.

MATHEMATICAL MODELING OF VISCOELASTIC DAMPERS

Figure 1 shows a schematic diagram of the shape of a VED and a structure installed with a VED. The damper dissipates vibrational energy through shear-type deformation of viscoelastic material which retains both elasticity and viscosity. Figure 2 describes the mathematical modeling of a structure with a VED unit used in the analysis, and the force–displacement relationship of a VED in the frequency domain is generally expressed as follows:

$$F_{\mathrm{d}}(\omega) = \{E_{\mathrm{s}}(\omega) + E_{\mathrm{l}}(\omega)\}\delta(i\omega) \tag{1}$$

where $E_{\mathrm{s}}(\omega)$ and $E_{\mathrm{l}}(\omega)$ are the storage and the loss stiffness of the damper, respectively. In this study the behavior of a VED is modeled by the Kelvin model, in which a linear spring and a dashpot are connected in parallel, and Equation (1) becomes

$$F(i\omega) = (k_{\mathrm{d}} + ic_{\mathrm{d}}\omega)\delta(i\omega) \tag{2}$$

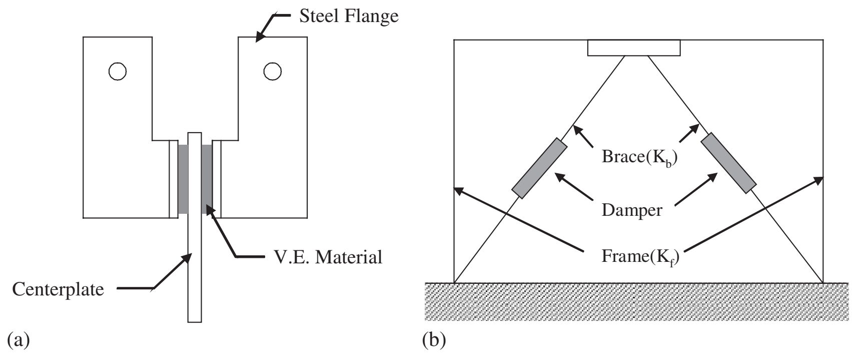

Figure 1. Schematic diagram of the shape of a VED and a frame with a VED: (a) VED; and (b) frame segment.

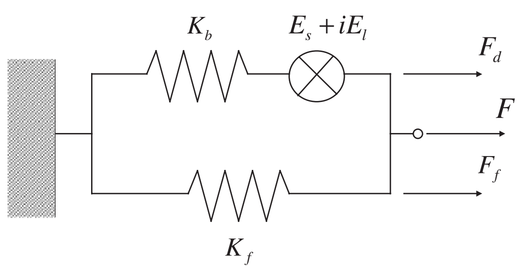

Figure 2. Mathematical model of a SDOF system with VED.

OPTIMUM DESIGN OF THE VED BY EIGENVALUE ASSIGNMENT

Eigenvalue assignment using the convex model

In this study the desired target displacement is determined first, and the natural frequency and the damping ratio of the structure are computed using the convex model, which is one of the methods of predicting maximum responses for non-stationary earthquake load [2]. The convex model is known to be useful especially when only a limited amount of information about the load exists, although the solution may become conservative. There are many variables to be used to represent the uncertainty of earthquake loads in the convex model, and the global energy-bound (GEB) convex model that uses the earthquake energy as the main variable will be used in this study. The following equation represents the maximum displacement corresponding to the i-th vibration mode by earthquake energy I_0, natural period T, damping ratio ξ, and the mode shape vector of the i-th mode ϕ_i:

$$S_{yi}(T_i, \xi_i) = \frac{\phi_i^{\mathrm{T}} I_0 \sqrt{E} T^{3/2}}{4\pi\sqrt{2\pi\xi}} \tag{3}$$

With the maximum modal displacements, the maximum displacement of the system can be obtained using the Square Root of the Sum of Squares method (SRSS):

$$|x_{\max,j}| \cong \sqrt{\sum_{i=1}^{N} \phi_{ij}^2 S_y^2(\omega_i, \xi_i)} \tag{4}$$

In this study it is assumed that the installation of a VED does not affect the acceleration response significantly, because the natural frequency of the structure increases only slightly by the addition of stiffness from the VED but decreases rather considerably due to the addition of damping from the damper. Also, as the safety of a structure depends primarily on the displacement response, the natural frequency and modal damping ratio of the structure will be assigned by focusing on the decrease of the maximum displacement response using the convex model. The proper combinations of natural frequency and modal damping ratio that can realize the given target maximum displacement or the rate of change in maximum displacement can be determined by using the following equation as well as Equation (3).

$$R = \frac{S_{yi}(T_i, \xi_i)}{S_{yi0}(T_{i0}, \xi_{i0})} = \frac{\phi_i^{\mathrm{T}} I_0}{\phi_{i0}^{\mathrm{T}} I_0} \left(\frac{T_i}{T_{i0}}\right)^{3/2} \sqrt{\frac{\xi_{i0}}{\xi_i}} \cong \left(\frac{T_i}{T_{i0}}\right)^{3/2} \sqrt{\frac{\xi_{i0}}{\xi_i}} \tag{5}$$

where R is the ratio of the displacement response before and after the dampers are installed, ξ_i, T_{i0} and S_{yi} are the damping ratio, natural period, and maximum displacement of the i-th mode obtained after the VEDs are installed, respectively, and those with a subscript zero are quantities associated with the original structure. It is assumed in the above equation that the mode shapes of the structure before and after the dampers are installed are the same. This assumption is reasonable since the stiffness of the VED is generally significantly small compared to that of the structure. Therefore the response reduction ratio (Equation (5)) can be obtained if the natural frequency and damping ratio of each mode are known. The derivative of the expression for maximum displacement (Equation (3)) with respect to the natural frequency and modal damping ratio leads to:

$$\frac{\partial S_{yi}}{\partial T_i} = \frac{\phi_i^{\mathrm{T}} I_0 \sqrt{E}}{4\pi\sqrt{2\pi\xi_i}} \frac{3}{2} T_i^{1/2} \tag{6}$$

$$\frac{\partial S_y}{\partial \xi_i} = \frac{\phi_i^{\mathrm{T}} I_0 \sqrt{E}}{4\pi\sqrt{2\pi}} T_i^{3/2} \cdot -\frac{1}{2} \xi_i^{-3/2} \tag{7}$$

The gradients of the maximum displacement (Equations (6) and (7)) are used to determine the optimum natural period and the damping ratio of the i-th mode that can realize the target displacement. When the derivative of the maximum displacement with respect to the natural period (Equation (6)) is larger than that with respect to the damping ratio (Equation (7)), to increase stiffness is more effective than to increase damping, and vice versa. In case the natural period is reduced excessively as a result of adding stiffness, however, the maximum acceleration may increase significantly even though the maximum displacement decreases. Also, as the present study assigns eigenvalues as well as damping ratios for the optimum design, the natural periods should not change significantly as a result of damper installation. Therefore in this study the gradient of the maximum displacement for the natural period is reduced by a proper reduction factor, and the reduction of natural periods after the dampers are installed is limited within a certain value:

$$\frac{\partial S_{yi}}{\partial T_i} = \gamma \left(\frac{\phi_i^{\mathrm{T}} I_0 \sqrt{E}}{4\pi\sqrt{2\pi\xi_i}} \frac{3}{2} T_i^{1/2} \right) \quad 0 \leqslant \gamma \leqslant 1 \tag{8}$$

$$\frac{T_i}{T_{i0}} \geqslant \kappa \quad 0 \leqslant \kappa \leqslant 1 \tag{9}$$

Also, the possible range of additional stiffness and damping which can be supplied by the VED is another factor for optimum design. The stiffness and damping of the VED are related, and the installation of the VED to increase system damping also increases the stiffness of the system. Therefore the eigenvalue and damping ratio should be assigned in such a way that the loss factor (the ratio of storage and loss coefficients of the VED) becomes the standard value for viscoelastic material, which is 1.2–1.4.

Optimal distribution of design parameters

The non-linear programming algorithm for determining control gains that can accomplish the desired eigenvalue is presented in Figure 3 [4]. The state–space form expression of the equations of motion of a structure with assigned natural frequency and modal damping ratio determined from the convex model is expressed as follows:

$$\begin{bmatrix} \dot{u} \\ \ddot{u} \end{bmatrix} = \begin{bmatrix} O & I \\ -K_{\mathrm{D}} & -C_{\mathrm{D}} \end{bmatrix} \begin{bmatrix} u \\ \dot{u} \end{bmatrix} \tag{10}$$

where $K_{\mathrm{D}} = \mathrm{diag}[\omega_{1\mathrm{D}}^2, \omega_{2\mathrm{D}}^2, \ldots, \omega_{n\mathrm{D}}^2]$, $C_{\mathrm{D}} = 2\mathrm{diag}[\omega_{1\mathrm{D}}\xi_{1\mathrm{D}}, \omega_{2\mathrm{D}}\xi_{2\mathrm{D}}, \ldots, \omega_{n\mathrm{D}}\xi_{n\mathrm{D}}]$, and $\omega_{i\mathrm{D}}$ and $\xi_{i\mathrm{D}}$ are the natural frequency and the modal damping ratio to be assigned, respectively, and n denotes the number of assigned eigenvalues. The eigenvalue of the state–space equation (Equation (10)) is expressed in complex numbers as follows:

$$\lambda_{i\mathrm{D}} = -\xi_{i\mathrm{D}}\omega_{i\mathrm{D}} \pm i\omega_{i\mathrm{D}}\sqrt{1 - \xi_{i\mathrm{D}}^2} \tag{11}$$

When the VED is modeled by the Kelvin model, the force–displacement relationship can be expressed as:

$$f(t) = k_{\mathrm{d}}\delta(t) + c_{\mathrm{d}}\dot{\delta}(t) \tag{12}$$

If N VED are installed the stiffness and damping coefficients of the VED can be expressed as the following parameter vector:

$$\begin{aligned} p &= [p_1, p_2, p_3, p_3, \ldots]^{\mathrm{T}} \\ &= [k_{\mathrm{d}1}, k_{\mathrm{d}2}, \ldots, k_{\mathrm{d}N}, c_{\mathrm{d}1}, c_{\mathrm{d}2}, \ldots, c_{\mathrm{d}N}]^{\mathrm{T}} \end{aligned} \tag{13}$$

The feedback state–space equation and control force are given as:

$$\dot{x} = Ax + Bu \tag{14}$$

$$u = -Gx \tag{15}$$

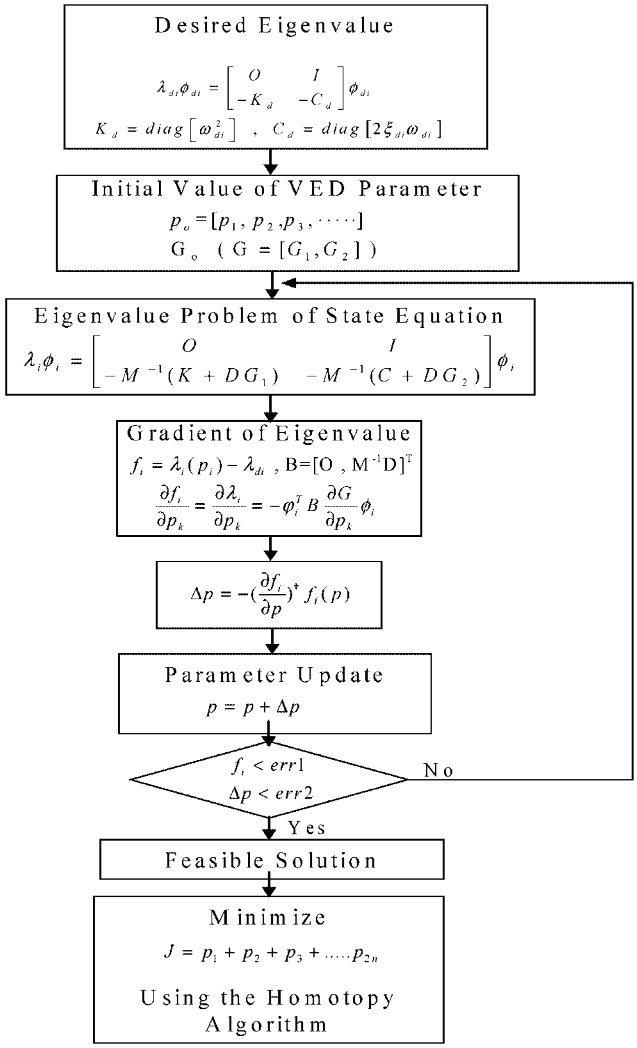

Figure 3. Algorithm for determining control gains.

where matrices A, B and the gain matrix G for the passive control system by the VEDs in a MDOF structure are given as:

$$A = \begin{bmatrix} O & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix}, \quad B = \begin{bmatrix} O \\ M^{-1}D \end{bmatrix} \tag{16}$$

$$G = [G_1 \;\; G_2] \tag{17}$$

where G_1 and G_2 are control gain matrices contributed from the stiffness and viscosity of the VEDs, respectively, and D is the influence matrix of VED. The state–space equation

(Equation (14)) can be transformed to Equation (18) and its eigenvalues can be obtained as Equation (19):

$$\lambda_i \phi_i = \begin{bmatrix} O & I \\ -M^{-1}(K + DG_1) & -M^{-1}(C + DG_2) \end{bmatrix} \phi_i \tag{18}$$

$$\lambda_i = -\xi_i \omega_i \pm i\omega_i \sqrt{1 - \xi_i^2} \tag{19}$$

where ω_i and ξ_i are the natural angular frequency and modal damping ratio, respectively, for a structure with added VEDs. Equation (20) is an assignment function of eigenvalues, f_i, which is defined as a difference between the current eigenvalues and the eigenvalues being assigned.

$$f_i = \lambda_i(p) - \lambda_{i\mathrm{d}} \tag{20}$$

$$\underline{f} = [f_1, f_2, f_3, \ldots, f_{2n}]^{\mathrm{T}} \tag{21}$$

The gradient of f_i with respect to p_k is given as:

$$\frac{\partial f_i}{\partial p_k} = \frac{\partial \lambda_i}{\partial p_k} = -\varphi_i^{\mathrm{T}} B \frac{\partial G}{\partial p_k} \phi_i \tag{22}$$

The updated parameter increment $\Delta \underline{p}$ is obtained from the Minimum Norm Correction Algorithm:

$$\underline{f}(\underline{p} + \Delta\underline{p}) = \underline{f}(\underline{p}) + \left[\frac{\partial \underline{f}}{\partial \underline{p}}\right]_p \Delta\underline{p} + 0\left(\frac{\partial \underline{f}}{\partial \underline{p}}\right) = 0 \tag{23}$$

$$\Delta\underline{p} = -\left[\frac{\partial \underline{f}}{\partial \underline{p}}\right]_p^{+} \underline{f}(\underline{p}), \quad \text{minimize norm } \|\Delta\underline{p}\| \tag{24}$$

$$\underline{p} = \underline{p} + \Delta\underline{p} \tag{25}$$

The gradient of $\underline{f}$ is calculated until the assigned eigenvalue function (f_i) converges to 0, thus updating the parameter. Then the Homotopy Algorithm [4] is adopted to minimize the sum of parameters associated with the VEDs and to assign eigenvalues determined using the minimum amount of the VEDs. The following temporary objective function (f_p) is defined with the Homotopy parameter (γ):

$$\underline{f}_p(\gamma) = \gamma \underline{f}_0 + (1-\gamma)\underline{f}(\underline{p}_{\mathrm{start}}); \; 0 \leqslant \gamma \leqslant 1, \; (\underline{f}_0)_{2N+1} = 0 \tag{26}$$

where f_p consists of the sum of eigenvalues and parameters, and f_0 is the final target value.

The minimum function ($\underline{H}$) is defined as:

$$\underline{H}(p(\gamma), \gamma) = \underline{f}_p(\gamma) - \underline{f}(p) \tag{27}$$

$$f_i = \lambda_i(p) - \lambda_{i\mathrm{d}} \quad i = 1, 2, 3, \ldots, 2n \tag{28}$$

$$f_{2N+1} = p_1 + p_2 + p_3 + \cdots + p_{2N} \tag{29}$$

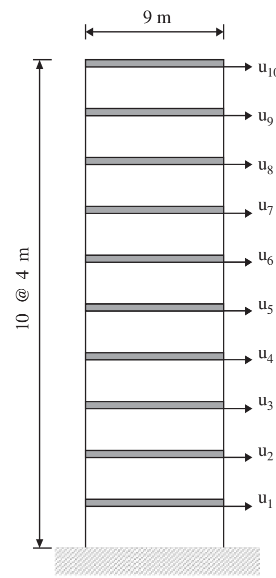

Figure 4. 10-Story plane frame model structure.

Then the parameter decrement is obtained by the following equation:

$$\Delta \underline{p} = -\left[\frac{\partial \underline{f}}{\partial \underline{p}}\right]_p^{+} H(p(\gamma), \gamma) \tag{30}$$

Whenever f does not converge to $\underline{f}_p(\gamma)$ within a specified number of iterations, the parameter Δp is updated by decreasing the Homotopy parameter γ. The iteration process stops in the case where γ is smaller than the specified value. The parameter vector (p) represents the optimal distribution of the minimum amount of the VEDs, and f_{2N+1} indicates the minimum amount of the VEDs used in the building. The algorithm for optimization of the VED parameters is presented in Figure 3.

APPLICATION TO PLANE-FRAME SHEAR BUILDING

Model structure for analysis

The optimum design of the VED for a 10-story shear building shown in Figure 4 was conducted using the eigenvalue assignment technique. The mass and stiffness of each story are given in Table I. The Rayleigh damping matrix, which provides 2% of the critical damping for every mode, is used. The modal participation factor for the first mode turned out to be 94%, which shows that the fundamental mode dominates the structural behavior.

Table I. Story mass and stiffness of the model structure.

Story	Mass (kg)	Story stiffness (kg/cm)
1	24300	10000
2	24300	8000
3	24300	8000
4	24300	8000
5	24300	7000
6	24300	7000
7	24300	7000
8	24300	5000
9	24300	5000
10	24300	5000

Gradient of modal properties

Figures 5 to 7 represent the gradients of the fundamental natural period and the modal damping ratio of the structure before the VEDs are installed with respect to the stiffness and damping parameters of the VED. It can be seen in Figure 5 that the gradients of the natural frequency for the fundamental mode with respect to added stiffness are relatively large in the lower stories and are small in the higher stories. At the 2nd, 5th, and 8th stories, where inter-story drifts are expected to be large, the gradients increase. The gradients of the fundamental modal damping ratio for added damping, plotted in Figure 7, show a similar trend. The following equations represent the natural frequency and its gradient with respect to the story stiffness when the assumed mode shape vector φ_i is used to transform the MDOF system to a SDOF system:

$$\omega_n^2 = \frac{\sum_{j=1}^{N} k_j(\varphi_j - \varphi_{j-1})^2}{\sum_{j=1}^{N} m_j \varphi_j^2} \tag{31}$$

$$\frac{\partial \omega_n^2}{\partial k_i} = \frac{(\varphi_i - \varphi_{i-1})^2}{\sum_{j=1}^{N} m_j \varphi_j^2} \tag{32}$$

In Equation (32) the gradient of the natural frequency is largest where the difference between the mode shape coefficients corresponding to each pair of neighboring stories is largest. In other words, to place a VED in the stories with larger inter-story drift is most effective in increasing the natural frequency and decreasing structural responses. Figure 5 coincides well with the observation that the gradient of natural frequency for damper stiffness is large where the inter-story drift is large. It is obvious to note that installing dampers in stories where large inter-story drift occurs will be most effective in reducing structural responses, and the results presented above verify that optimum design can be achieved by using the information related to the gradient of the design parameters of the VED.

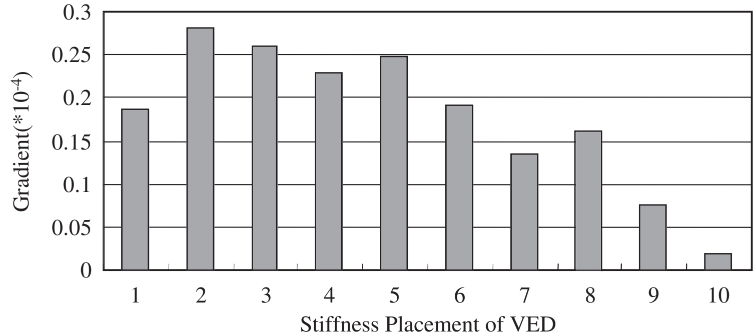

Figure 5. Gradient of the 1st mode natural frequency with respect to stiffness of the VED.

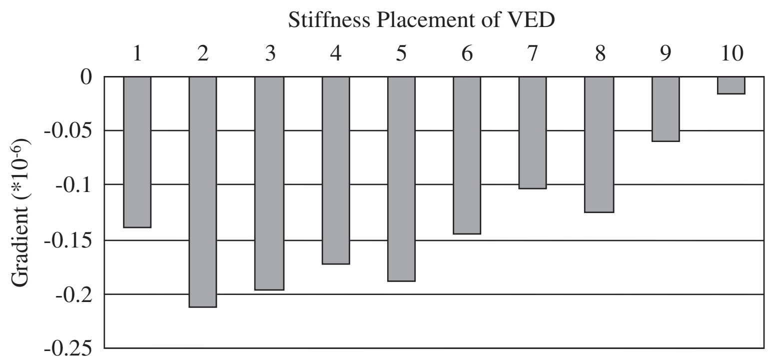

Figure 6. Gradient of the 1st modal damping ratio with respect to stiffness of the VED.

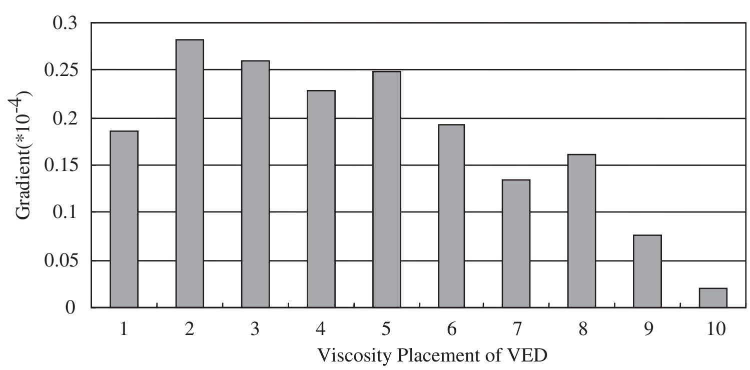

Figure 7. Gradient of the 1st modal damping ratio with respect to viscosity of the VED.

Figure 6 shows that the gradient of the damping ratio of the fundamental mode with respect to the stiffness of the added VED is negative in every story; i.e. the modal damping ratio decreases as the stiffness of the structure increases. The magnitude of the gradient, however, is very small compared to that with respect to the added damping. Therefore it can be concluded that the stiffness from the added VED does not affect the modal damping ratios of the structure significantly.

Table II. Eigenvalues for the fundamental mode to be assigned.

	Original structure	Assigned values
Natural frequency	0.4238 Hz	0.4431 Hz
Modal damping ratio	2%	7%

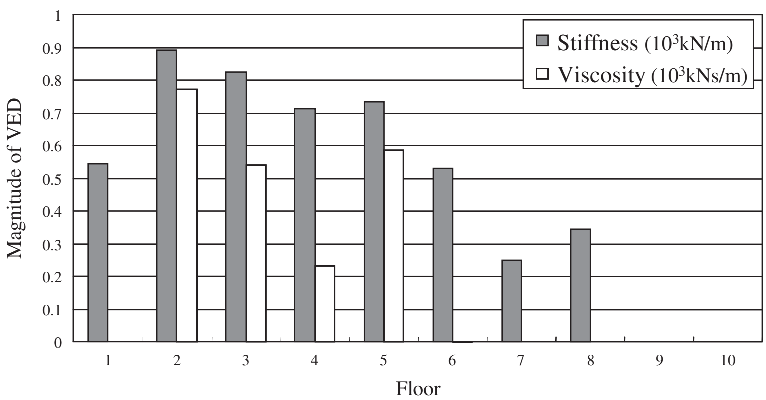

Figure 8. Distribution of stiffness and viscosity of the VED.

Response of the model structure

As the dynamic behavior of the model structure is dominated by the fundamental mode, modal properties associated only with the first mode are assigned. Table II presents the natural frequency and the modal damping ratio of the fundamental mode obtained from Equation (5) to reduce the displacement response by 50%. Figure 8 shows the optimum distribution of the stiffness and damping of the VED obtained using the assigned modal properties, where it can be found that the story-wise distribution of the damper parameters is similar to that of the gradient of the natural frequency with respect to the parameters. This is because the distribution of damper parameters is governed by the gradient information obtained through iterations, which are initially started with the gradient of building. It can be noticed, however, that the damper stiffness is distributed to every story, whereas the allocation of damping property is concentrated on the 2nd to 5th stories in which the gradients of the modal damping ratio are relatively large. This is due to the fact that in the process of minimizing the damper parameters the re-distribution process is focused on the damping property because changing damping is more effective in reducing displacement than changing stiffness.

In case the optimum stiffness and damping allocated to each story are to be provided by the VEDs, only those stiffness–damping combinations corresponding to a proper VED loss factor, inherent in the viscoelastic material used, can be effective. If the stiffness of the VED is smaller than the specified optimum value, additional bracing can be installed parallel to the VED to satisfy the required stiffness obtained from the eigenvalue assignment method.

Table III. Eigenvalues before and after the installation of the VEDs.

Mode	Natural frequency (Hz)		Modal damping ratio (%)	
	Before	After	Before	After
1	0.4238	0.4431	2.00	7.00
2	1.1686	1.9993	2.00	7.59
3	1.9208	1.9733	2.00	15.39
4	2.6645	2.7254	2.00	13.10
5	3.2537	3.3500	2.00	15.98
6	3.8572	3.9419	2.00	28.00
7	4.2416	4.3403	2.00	22.70
8	4.7219	4.8834	2.00	35.47
9	5.1402	5.3460	2.00	65.34
10	5.5681	5.8344	2.00	100.00

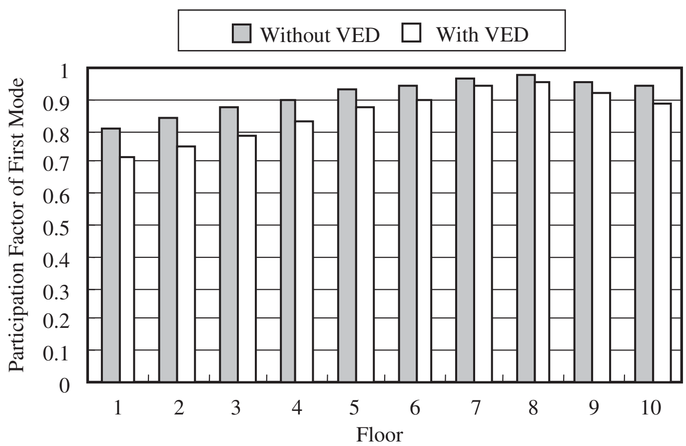

Figure 9. Participation factor for the 1st mode.

Table III and Figure 9 show the change in the natural frequencies and the participation of the fundamental mode before and after the dampers are installed, respectively. As only the fundamental modal properties were assigned, the natural frequencies and the modal damping ratios for higher modes can have arbitrary values. However, it can be observed in Table III that after the dampers are installed the higher-mode modal damping ratios become larger than those of the fundamental mode. This is natural considering the fact that the VED is modeled by the Kelvin model, in which the loss factor is proportional to the natural frequency of the structure. However, as the participation of the fundamental mode is more than 70% of all modes even after the dampers are installed, the optimum design by allocation of only the fundamental modal property seems to be reasonable. Viscosity of the VEDs is determined by the viscous coefficient c_d in the process of eigenvalue assignment. As this method adopts the Kelvin model in which the loss coefficient increases proportionally to loading frequencies, higher-mode damping ratios turn out to be at least 7% larger than those of the first mode after the installation of the VEDs. Therefore, when VEDs are modeled by the Kelvin model and the proposed optimum design method is applied, the reduction in the higher-mode responses

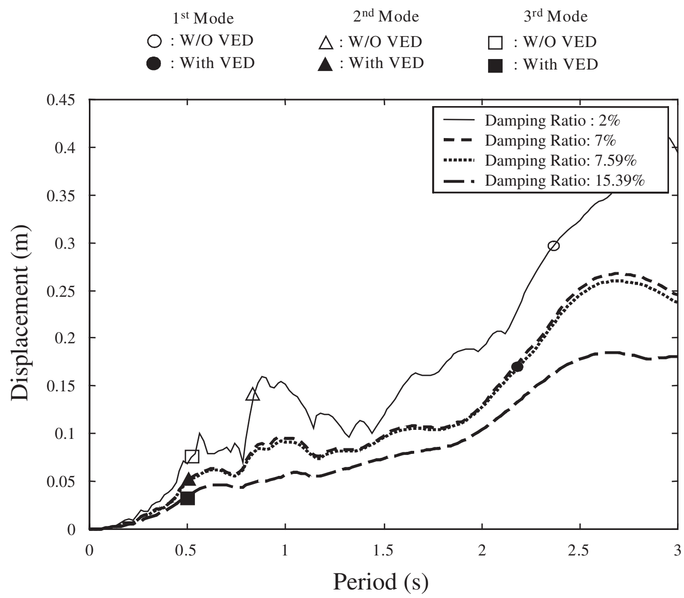

Figure 10. Displacement response spectra of the model structure for the El Centro earthquake.

Table IV. Combinations of natural frequency and damping ratio for the fundamental mode.

	Bare frame	Case 1	Case 2	Case 3
Natural frequency (Hz)	ω_0	1.071 ω_0	1.046 ω_0	1.022 ω_0
	0.4238	0.4542	0.4431	0.4330
Damping ratio (%)	2.0	6.5	7.0	7.5

will be larger than those of the first-mode response. However, as can be seen in Figure 10, the participation of the first mode in every story response is higher than 70% after the installation of the VEDs, which indicates that the proposed optimal design method using eigenvalue assignment can be valid.

Figure 10 shows the displacement response spectra of the model structure subjected to the El Centro earthquake, and the responses corresponding to the first three vibration modes are also marked on the figure. It can be observed that the modal responses of the structure, with the fundamental modal properties determined from the convex model to achieve 50% reduction of responses, are reduced almost to half also for the El Centro earthquake.

Comparison of the effect of change in assigned modal properties

To investigate the effect of change in the assigned modal properties on the change in the optimum damper properties and total amount of dampers, the results for the three combinations of assigned natural frequency and modal damping ratio, computed using the convex model, were compared. Each combination of modal properties is presented in Table IV. All three

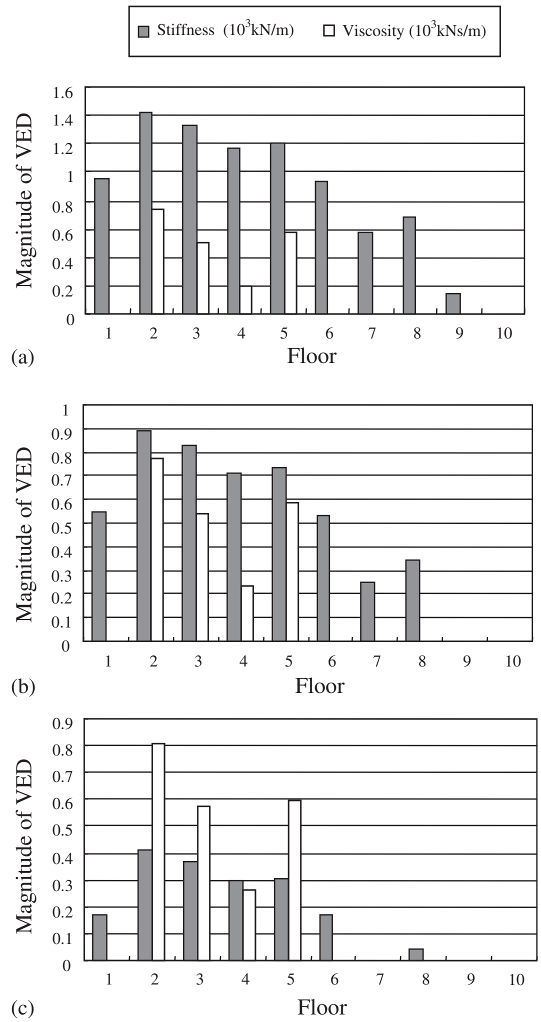

Figure 11. Distribution of stiffness and viscosity of the VED: (a) Case 1; (b) Case 2; and (c) Case 3.

cases correspond to eigenvalues which reduce the maximum displacement by 50%. Figure 11 demonstrates that as the assigned natural frequency and modal damping ratio increase, greater damper stiffness and damping coefficient are required. It can be observed that the overall proportion of story-wise distribution of damper properties is similar in every case, even though the required amounts of stiffness and damping in each story change. It also can be noticed, however, that for a small change in natural frequency the stiffness changes very rapidly, whereas damping changes only slightly. This implies that the natural frequency is not sensitive to the change in damper stiffness. Therefore it would be reasonable to determine the optimum size of the VED based primarily on the damping of the VED.

Table V presents root mean squared (RMS) responses of the model structure obtained from the frequency domain analyses before and after the dampers are installed. The power spectral

Table V. Results of the frequency domain analysis.

	Bare frame	Case 1	Case 2	Case 3
Max. inter-story drift (cm)	9.1552	4.3718	4.4255	4.4861
Max. displacement (cm)	69.6710	35.1139	35.2781	35.3961
Max. acceleration (m/s^2)	28.8803	28.5063	28.4999	28.4941

Table VI. Frequency dependent ground acceleration.

Frequency	ξ_g	ω_g	ξ_f	ω_f
Low frequency range	2π	0.40	0.20π	0.40
Wide frequency range	10π	0.80	π	0.80

density function of the input ground acceleration [6], which is the Kanai–Tajimi spectrum multiplied by the transfer function of the foundation of the structure, is as follows:

$$S_{\ddot{x}_g}(\omega) = S_0 \left[\frac{1 + 4\xi_g^2(\omega/\omega_g)^2}{[1-(\omega/\omega_g)^2]^2 + 4\xi_g^2(\omega/\omega_g)^2} \right] \left[\frac{(\omega/\omega_f)^4}{[1-(\omega/\omega_f)^2]^2 + 4\xi_f^2(\omega/\omega_f)^2} \right] \tag{33}$$

where $S_0 = 100$, and the variables ω_g, ξ_g are determined by soil condition, and ω_f, ξ_f are from the type of foundation. Table VI lists the value of the variables used in the analysis [1]. It can be observed that in the three cases for damper installation the responses are reduced almost to half. Although not very distinct, it can be noticed in the results of the three cases that when the damper stiffness is larger the reduction in displacement is more significant than acceleration. The opposite is true when the damping is larger.

APPLICATION TO PLAN-WISE ASYMMETRIC STRUCTURE

Asymmetric model structure

Figure 12 shows the plan-wise asymmetric structure for application of the proposed method. The model structure, which is 10-stories high with 4 m inter-story height, and is 9 m wide in both directions, has three DOFs; translations for the X and Y directions, and a rotation around the vertical axis. For simplicity, the asymmetry is assumed to exist only along the Y-axis. Table VII presents the mass and stiffness in each story of the model structure. The story stiffnesses of Frames A and B are identical to those of the symmetric model structure used previously. However, the story stiffness in each story of Frame C is 1.5 times larger than that for the symmetric model, although the summation of stiffness of Frames C and D remains the same. The modal damping ratios are given to be 2% for every vibration mode.

Table VIII shows the modal participation factors for the translational and rotational responses of the model structure, where it can be observed that for responses in both directions the participation of the first and second modes dominate. Therefore, in this study only the eigenvalues for the first and second modes are assigned for optimum application of the VED in the plan-wise asymmetric structure.

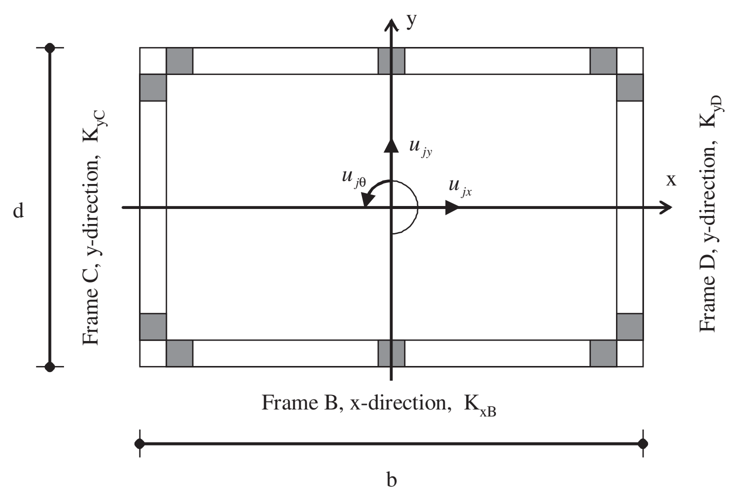

Figure 12. 10-Story plan-wise asymmetric model structure.

Table VII. Properties of the asymmetric model structure.

Story	Mass (kg)	Story stiffness (kg/cm) (Frame A, B)	Story stiffness (kg/cm) (Frame C)	Story stiffness (kg/cm) (Frame D)
1	24300	5000	4000	6000
2	24300	4000	3200	4800
3	24300	4000	3200	4800
4	24300	4000	3200	4800
5	24300	3500	2800	4200
6	24300	3500	2800	4200
7	24300	3500	2800	4200
8	24300	2500	2000	3000
9	24300	2500	2000	3000
10	24300	2500	2000	3000

Table VIII. Participation factors for the first two modes.

	Translational response		Torsional response	
Story	1st mode (%)	2nd mode (%)	1st mode (%)	2nd mode (%)
1	80.79	0.38	61.48	19.69
2	84.22	0.39	64.08	20.53
3	87.24	0.41	66.38	21.26
4	89.79	0.42	68.33	21.89
5	92.44	0.43	70.34	22.53
6	94.10	0.44	71.61	22.94
7	96.28	0.45	73.26	23.47
8	96.71	0.45	73.59	23.57
9	95.32	0.44	72.53	23.23
10	93.66	0.44	71.27	22.83

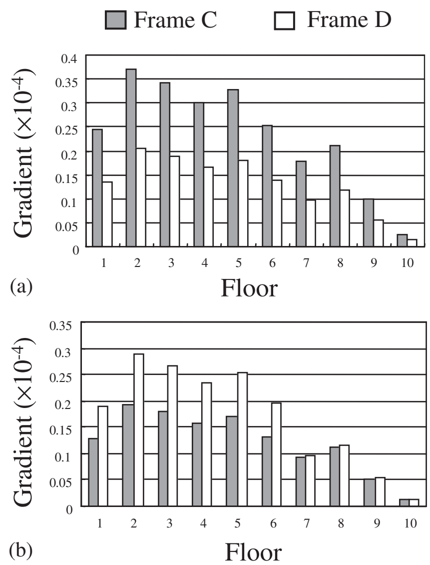

Figure 13. Gradient of natural frequency with respect to stiffness of the VED: (a) 1st Mode; and (b) 2nd Mode.

Gradient of modal properties

Figure 13 presents the gradient of natural frequency with respect to damper stiffness, and Figure 14 plots the gradient of modal damping ratio for damper viscosity. The results show that for the modal properties of the first mode, the gradients of Frame C with smaller stiffness are larger than those of Frame D with larger stiffness.

This implies that to install the dampers in Frame C is more effective in increasing the natural frequency and modal damping ratio for the first mode. Therefore the proposed method of optimum distribution of dampers is performed in such a way that the asymmetry of the model structure is reduced. It also can be noticed, however, that the gradients of modal properties of the second mode for Frame D are larger than those for Frame C; i.e. to install the VED in the stiffer frame results in a larger increment in modal properties of the second vibration mode. Even though this may signify the asymmetry owing to the addition of stiffness, the effect of added damping and the interaction with the first mode decreases the torsional responses. Therefore, in a plan-wise asymmetric structure it would be necessary to take both the translational and rotational vibration modes into account to determine the optimum damper parameters.

Figure 15 shows the schematic diagram of the first and the second relative modal displacement shapes of the roof floor of the model structure. It can be observed that in the first mode the deformation of Frame C is larger than that of Frame D, whereas in the second mode the opposite is true. Therefore it can be concluded that the story-wise distribution of gradients shown in Figures 13 and 14 is reasonable.

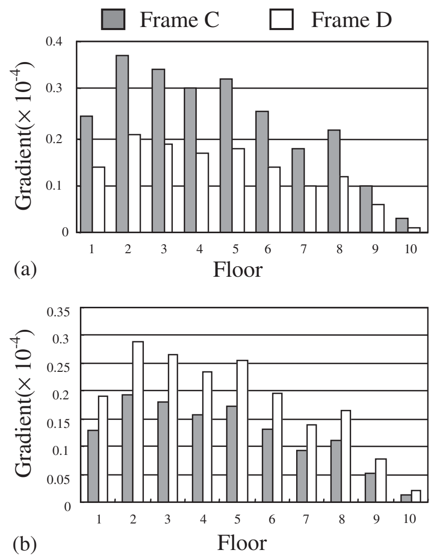

Figure 14. Gradient of modal damping ratio with respect to viscosity of the VED: (a) 1st Mode; and (b) 2nd Mode.

Response of the model structure

From the results of the convex model, the natural frequency and modal damping ratio are assigned in such a way that the displacement of each mode is reduced by 50%. Figure 16 shows the optimum distribution of stiffness and damping of the VED, respectively, when only the natural frequency and damping ratio of the first mode are assigned. As can be seen in the figures, when only the eigenvalue of the first mode is assigned, the damper stiffness installed in Frame C, which is the flexible edge, is twice as large as that which needs to be placed in Frame D, whereas almost all the added damping is allocated to Frame C. This implies that when only the first modal properties are assigned the dampers are placed in such a way that the plan asymmetry is reduced. Figure 17 depicts the transfer functions of the top-story displacements before and after the VED are installed. It can be found that the maximum peaks occur at the first-mode natural frequency which is the dominant mode for both translation and rotation, and that the magnitude of the transfer function is significantly reduced as a result of damper installation.

Next the damper properties were first distributed based on the assignment of the eigenvalue of the first mode, and then redistributed by assigning the eigenvalue of the second mode. In this case it can be observed in Figure 18 that the difference in the amount of the VED placed in Frames C and D is reduced. The transfer function for the top-story displacement, presented in Figure 19, shows results similar to those obtained for eigenvalue assignment of the first mode (Table IX).

Table X provides the analysis results in the frequency domain and the change in dynamic characteristics of the model structure after the optimally designed dampers are installed. The second and the third columns of the table denote the results for symmetric

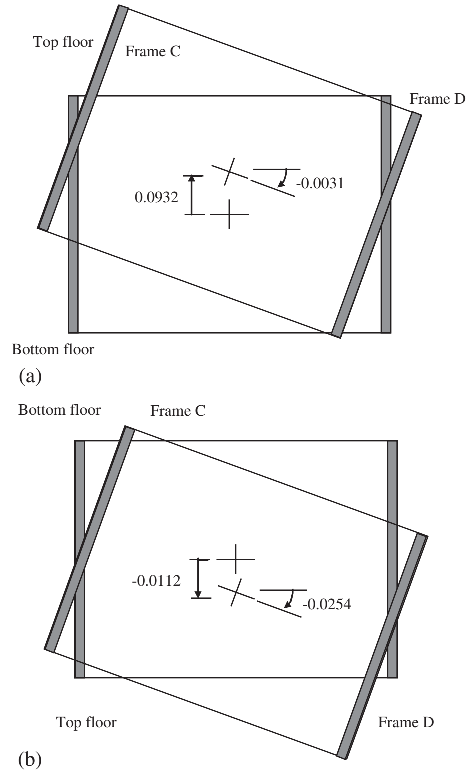

Figure 15. Modal deflection shape: (a) 1st Mode; and (b) 2nd Mode.

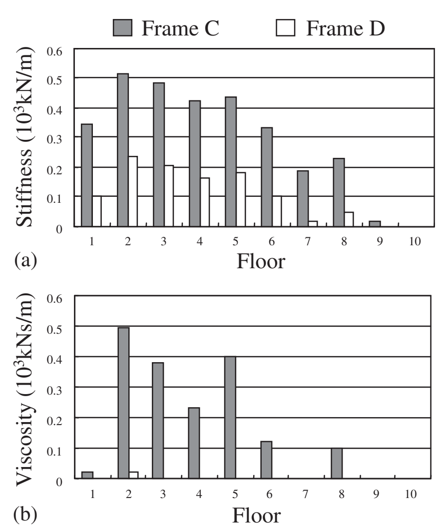

Figure 16. Distribution of VED parameters (only the eigenvalue of the first mode is assigned): (a) stiffness of the VED; and (b) viscosity of the VED.

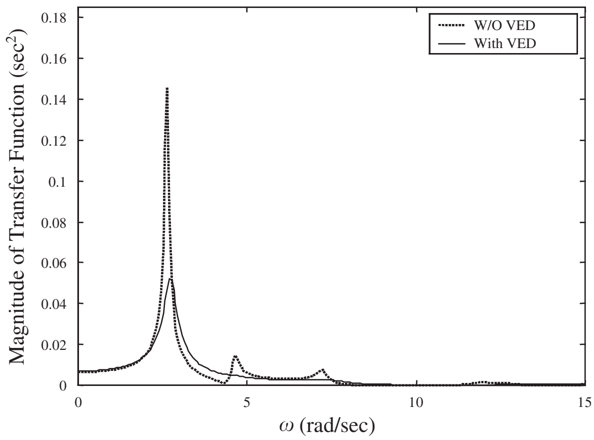

Figure 17. Transfer function of the top floor displacement of Frame C to the base input acceleration (only the eigenvalue of the first mode is assigned).

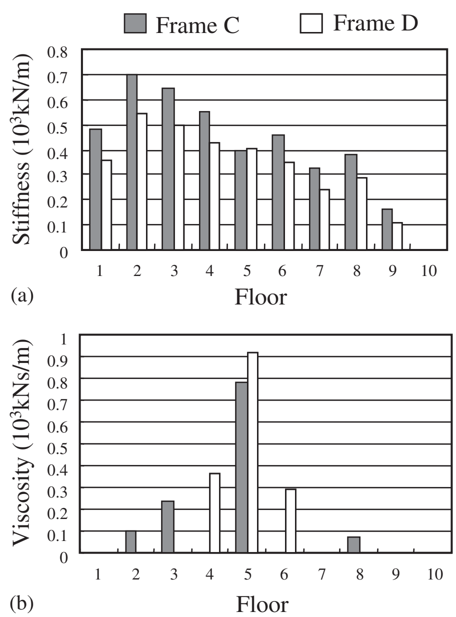

Figure 18. Distribution of VED parameters (both the eigenvalues of the first and the second modes are assigned): (a) stiffness of the VED; and (b) viscosity of the VED.

(X-direction) and asymmetric (Y-direction) model structures, respectively, before the dampers are installed. It can be observed that in the asymmetric structure the natural frequency of the first mode decreases and the maximum top-story displacement increases compared to those of the symmetric structure. When the natural frequency of the first mode (which is determined in advance to reduce the modal displacement of the first mode by 50%) is assigned the maximum displacement and the maximum inter-story drift decreased by 49% and 50%, respectively.

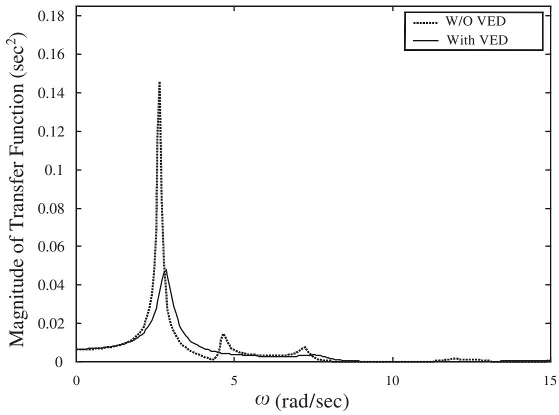

Figure 19. Transfer function of the top floor displacement of Frame C to the base input acceleration (both the eigenvalues of the first and the second modes are assigned).

Table IX. Eigenvalues to be assigned.

Eigenvalues	Bare frame	Assigned values
Natural frequency (1st mode)	0.4175 Hz	0.4365 Hz
Damping ratio (1st mode)	2%	7%
Natural frequency (2nd mode)	0.7376 Hz	0.7712 Hz
Damping ratio (2nd mode)	2%	7%

Table X. Modal characteristics and frequency-domain analysis results.

Responses and modal property	X-direction (symmetric)	Y-direction (asymmetric)	Assignment of 1st mode	Assignment of 1st and 2nd modes
Max. story drift (cm)	9.1552	11.3335	5.5628	5.3246
Max. disp. (cm)	69.6710	86.3001	43.3481	40.6562
Natural frequency (1st mode)	0.4238 Hz	0.4175 Hz	0.4365 Hz	0.4507 Hz
Damping ratio (1st mode)	2%	2%	7.00%	7.31%
Natural frequency (2nd mode)	—	0.7376 Hz	0.7515 Hz	0.7712 Hz
Damping ratio (2nd mode)	—	2%	5.01%	7.00%

When both the first and second mode natural frequencies are assigned, the maximum top-story and inter-story displacements further reduce to about 47%. It is interesting to note that when both eigenvalues are assigned the natural frequency of the first mode is also increased by about 7.3% because more dampers are needed to fix the dynamic characteristics of the structure to the assigned value.

CONCLUSIONS

This study presents a procedure for the optimum design of the VED by assigning eigenvalues required to achieve the desired structural response. The natural frequencies and modal damping ratios, required to realize a given target response, are determined first by the convex model. Then those modal properties are achieved by optimally distributing the damping and stiffness coefficients of the VEDs using non-linear programming based on the gradient of eigenvalues. This optimization method provides information on the optimal location as well as the magnitude of the damper parameters to achieve a given target.

Through the numerical analyses of a 10-story shear building and a plan-wise asymmetric structure, it was found that using the proposed method the damper parameters were mostly allocated to the inter-story with large drift or to the flexible edge, because the gradients of eigenvalues for damper parameters were larger in those places. This implies that the proposed method can provide a reasonable distribution of the VED in structures with a symmetric or an asymmetric plan to meet a given target displacement.

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