

# Adaptive linear analysis for inelastic seismic design of reinforced concrete moment frames

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**An adaptive linear analysis method for inelastic seismic design of reinforced concrete moment frames is developed adopting the well-known concept of incremental linear approximation for non-linear analysis. A series of linear analyses are performed for multiple lateral loading steps. After performing the linear analysis for each loading step, the analysis model of the structure is modified for the linear analysis of the next loading step addressing the current distribution of plastic hinges. By simply summing up the results of all piecewise linear analyses, the inelastic force and deformation demands of the members are directly determined. The proposed method is applied to the inelastic seismic design of regular and irregular reinforced concrete special moment frames and the design results are verified by comparing with the results of non-linear analysis. The adaptive linear analysis, which is aimed at application to the preliminary seismic design where the non-linear analysis is not preferred, can directly account for the effects of inelastic behaviour such as plastic mechanism of structure, moment redistribution between members and plastic deformations of members.**

## Introduction

Elastic analysis is common in preliminary seismic design, in spite of its technical inaccuracy, in determining the design forces and deformations of the members and structures. For the elastic analysis, as shown in Figure 1, the effective stiffness  $E_c I_e$  corresponding to the yield points (A in Figure 1) of the structures and members is used in most design codes including ACI 318-08 (ACI, 2008) and KBC 2009 (AIK, 2009). However, the actual seismic performances of the structures and members are related to points B (Figure 1), rather than the yield points A, because the structures and members are subjected to inelastic deformations. Therefore, it might be difficult for the elastic analysis to assure seismic safety of the structures subjected to inelastic deformations.

When a reinforced concrete (RC) moment frame is subjected to a strong earthquake, the forces and deformations of the structure and its members are significantly affected by the inelastic effects such as the plastic mechanism of the structure, moment redistribution between members and plastic deformations of members. For example, if the plastic deformation demand of a member is greater than the given deformation capacity, premature failure may occur and thus the target ductility of the structure may not be achieved. Such plastic deformation demand substantially

depends on the plastic mechanism of the structure and the moment redistribution between members. The design forces of the members are also significantly affected by the moment redistribution. Unlike the elastic moments with double curvature, the inelastic moments with single curvature occur in the columns at lower storeys owing to the moment redistribution. As a result, premature failure can occur involving early flexural yielding of columns and the resultant soft-storey mechanism.

To address the effects of the inelastic behaviour, secant stiffness methods have been studied for inelastic seismic design (Park and Eom, 2005; Priestley, 2000). In the secant stiffness methods, linear analysis using secant stiffness was adopted to address the effects of inelastic behaviour. Priestley (2000) idealised the structure with a conventional frame model which is the same as that used in elastic analysis. The secant stiffness of beams is then determined by dividing the elastic effective stiffness  $E_c I_e$  by the target ductility of the structure, considering the strong column–weak beam concept. However, in Priestley's method, plastic deformations of the members are not computed. On the other hand, in Park and Eom's method (Park and Eom, 2005), the plastic deformation and the moment redistribution of the members can be predicted by using the secant stiffness defined independently at each plastic hinge. However, this method

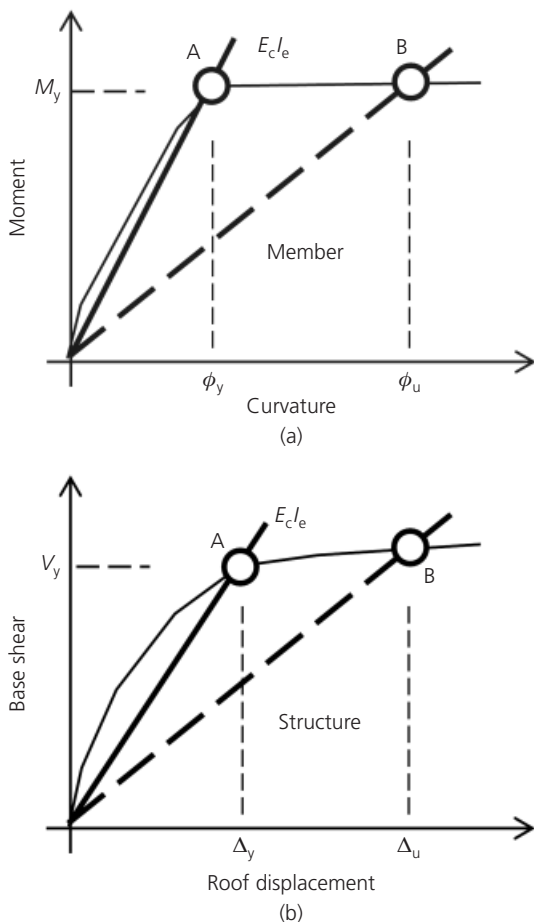


Figure 1. Elastic stiffness and secant stiffness: (a) moment–curvature relationship; (b) base shear–roof displacement relationship

requires significant efforts for modelling and iterative computation.

In the present study, an adaptive linear analysis method for inelastic seismic design was developed. Through non-linear analysis on a RC moment frame, the lateral load transfer mechanisms of the moment frame during the inelastic behaviour were investigated. Based on the result, the adaptive linear analysis procedure, which can account for the plastic mechanism of the structure, moment redistribution between members and plastic deformations of members, was developed. The proposed method was applied to the inelastic seismic design of regular and irregular moment frames and the design results were verified by non-linear analysis.

### Lateral load transfer mechanisms in moment frame

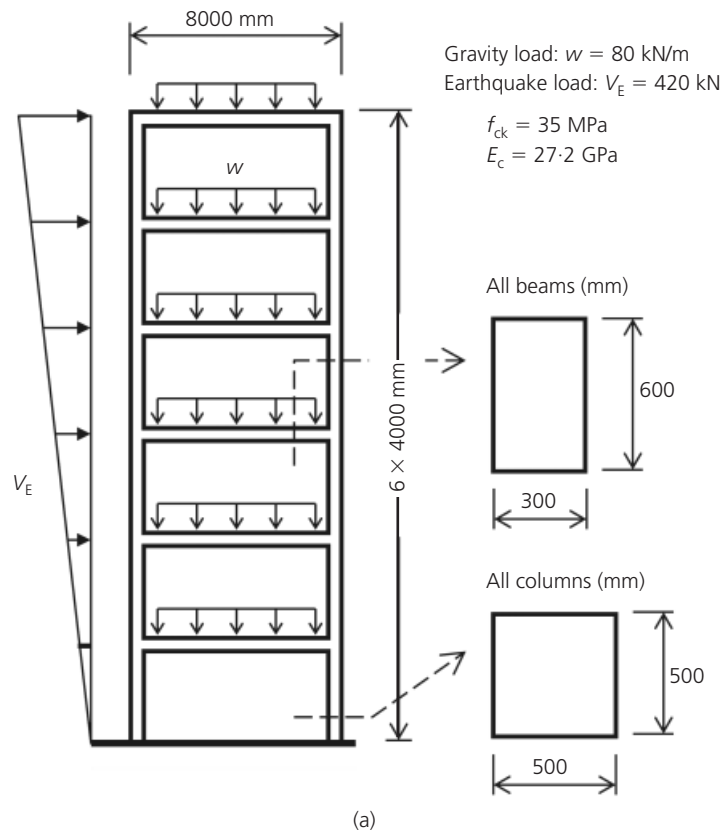
In this section the lateral load transfer mechanisms in a moment frame deforming in elastic and inelastic ranges were investigated. Figure 2(a) shows the configuration and loading condition of a

two-dimensional six-storey RC moment frame subjected to a lateral load. Its storey height and bay span are 4000 mm and 8000 mm, respectively. The cross-sections of the beams and columns are 300 mm  $\times$  600 mm and 500 mm  $\times$  500 mm, respectively. The effective stiffness values of the beams and columns are  $2 \times 0.5 E_c I_g$  (including slabs) and  $0.7 E_c I_g$ , respectively, where  $E_c$  and  $I_g$  denote the elastic modulus of concrete and the second-order moment of inertia of the gross section, respectively (ASCE, 2000). For seismic design of the model structure, the following three load combinations were considered: a gravity load case ( $1.4D + 1.7L$ ) and two cases with earthquake load ( $0.75 \times (1.4D + 1.7L) \pm 1.0E$ ), where  $D$ ,  $L$  and  $E$  designate the dead, live and earthquake loads, respectively.

The moment frame shown in Figure 2(a) was designed to satisfy the requirements on the special moment frame specified in ACI 318-08. The sum of the moment strengths of the columns connected to a joint was 1.2 times greater than that of the beams framing into the joint (ACI, 2008). Plastic deformations were allowed to develop at beams while the yielding of columns was avoided. Thus, the strong column–weak beam concept was achieved. At the beam ends, the positive moment strength (bottom bars in tension) of the cross section is greater than one-half of the negative moment strength (top bars in tension). The design moments of the beams and columns in the moment frame are shown in Figure 2(b). For the determination of the design moments, the direct inelastic design analysis (Park and Eom, 2005), which automatically incorporates the above requirements, was used.

To investigate the lateral load transfer mechanism in moment frames, a non-linear analysis (pushover analysis) was performed for the moment frame shown in Figure 2, by using Drain-2DX (Prakash *et al.*, 1993). For the non-linear analysis, the moment strengths shown in Figure 2(b) were used as the yield strengths of the beams and columns. When plastic hinges developed at the members, the plastic hinges were assumed to behave as elastic–perfectly plastic. Figure 3(a) shows the pushover curve of the moment frame in which points A to C represent the development of a sudden change in the stiffness and moment distribution due to the plastic hinge formation. As shown in Figure 3(a), the plastic hinges developed at the right ends of all the beams during the loading step O–A. During the loading step A–B, the plastic hinges developed at the left ends of the second to fourth floor beams. The plastic hinges then developed at the left ends of the fifth to the roof floor beams and the bottom of the first-storey columns during the loading step B–C. Finally, at point C, a complete plastic mechanism developed in the moment frame.

In order to analyse the variation in the lateral load transfer mechanism of the moment frame, the bending moment increments at the loading steps O–A, A–B and B–C are shown in Figures 3(c) to 3(e), respectively. Figure 3(b) shows the bending moment increment due to gravity load. Figure 3(c) shows the elastic bending moment increment at the loading step O–A

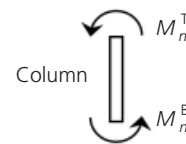
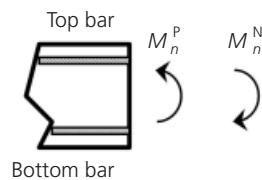


Beams

Floor	$M_u^P$	$M_u^N$
2	327	651
3	420	834
4	400	791
5	315	627
6	232	463
Roof	161	322

Columns

Storey	$M_u^B = M_u^T$
1	869
2	616
3	478
4	467
5	404
6	386



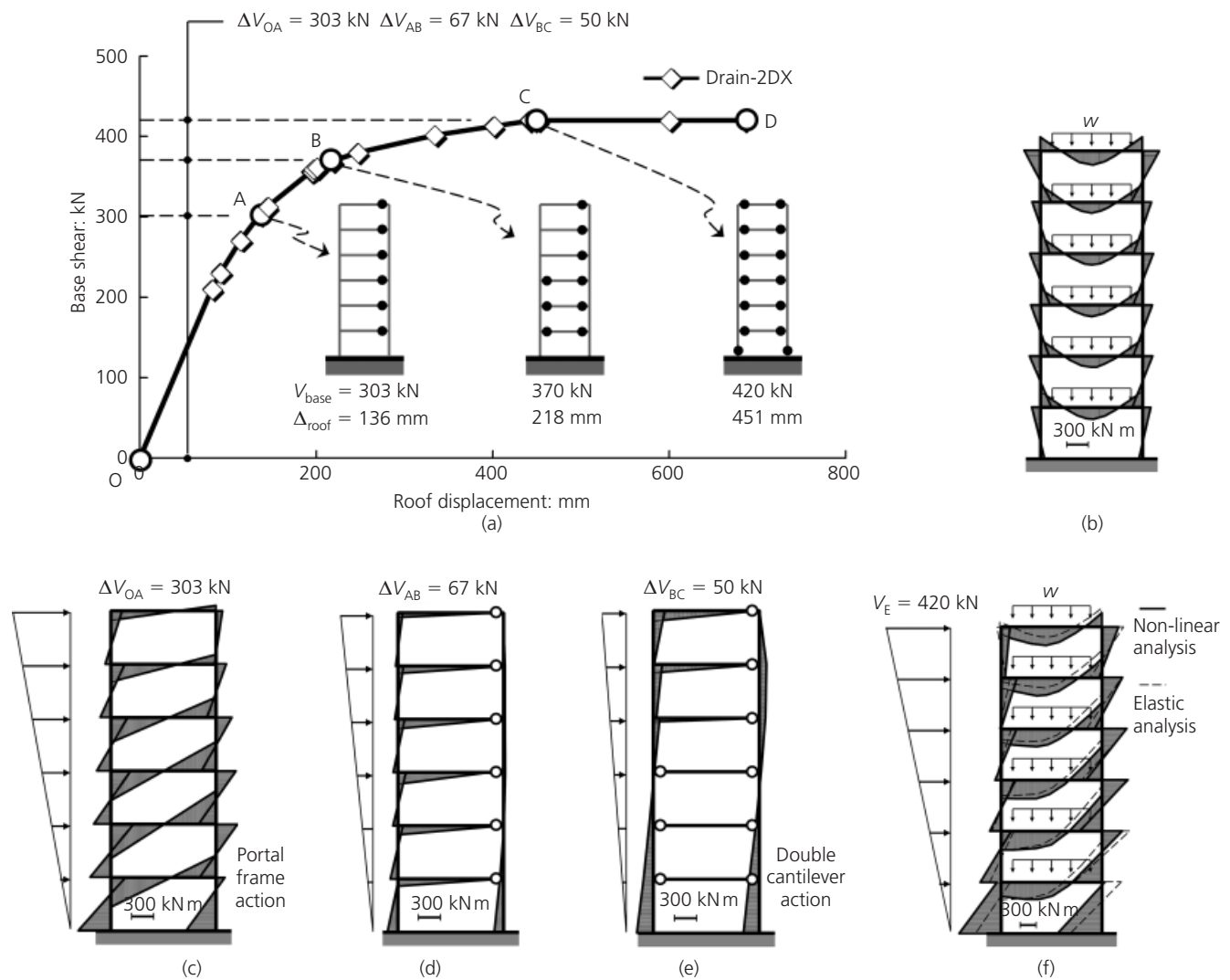
(b)

Figure 2. Configurations and design results of six-storey moment frame: (a) properties of six-storey moment frame; (b) design moments of beam and columns

( $\Delta V_{OA} = 303$  kN). Bending moments with double curvature developed at all the members, as frequently observed in the results of elastic analysis. Figure 3(d) shows the bending moment increment at the loading step A-B ( $\Delta V_{AB} = 67$  kN). Since the plastic hinges had already developed at the right ends of the beams at point A, the right ends of the beams could not transfer

the additional bending moments, and a small amount of the bending moment with single curvature along the building height occurred at the right columns.

At the left columns, on the other hand, the bending moments with double curvature developed owing to the remaining flexural



**Figure 3.** Non-linear analysis of six-storey moment frame designed by strong column-weak beam concept: Pushover curve and bending moment increments: (a) base shear–roof displacement relationship; (b) gravity load; (c) loading step O–A; (d) loading step A–B; (e) loading step B–C; (f) loading step O–A–B–C

stiffness of the beams. Figure 3(e) shows the bending moment increment at the loading step B–C ( $\Delta V_{BC} = 50 \text{ kN}$ ). Since the plastic hinges developed at both ends of the second to fourth floor beams at point B, a double cantilever action occurred spanning the first to fifth storey columns. Figure 3(f) shows the final bending moments of the moment frame due to the inelastic behaviour, calculated by summing the bending moment increments at all the loading steps O–A–B–C.

As shown in Figure 3, the distribution of the inelastic member moments in the moment frame designed by the strong column-weak beam concept has the following characteristics. Bending moments with single curvature occurred in the lower-storey

columns – double curvature, in the middle-storey columns, and single curvature, in the upper-storey columns. In the lower-storey columns, the bending moment at the bottom end was greater, but in the upper-storey columns, the bending moment at the top end was greater (Figure 3(f)). This moment distribution of columns mainly resulted from the double-curvature bending moment of the columns spanning the building height at the loading step B–C (double cantilever action in Figure 3(e)).

The negative bending moments at the ends of the beams were generally less than those obtained from the elastic analysis (Figure 3(f)). However, the bending moments of the lower-storey columns and the positive moments of the upper-storey beams

increased. This resulted from the fact that the plastic hinges developed early at the right ends of the beams; consequently, the bending moments that developed owing to further loading were redistributed to the left ends of the beams and to the lower/upper-storey columns that retained greater stiffness, as shown in Figures 3(d) and 3(e).

Such characteristics of the moment redistribution appear more clearly when greater plastic deformations are required to the moment frames due to premature yielding of the members. This result supports the requirements of ACI 318-08 (ACI, 2008), which specifies the increase in the moment strength of the columns and the positive moment strength of the beams for the special moment frame. However, it should be noted that the code requirements based on the results obtained from the elastic analysis may not ensure the seismic safety of structures.

### Adaptive linear analysis for inelastic seismic design

As shown in Figure 3(a), the load–displacement relationships of the structures experiencing inelastic deformations are non-linear in nature. However, if the major events of the plastic mechanism resulting from the non-linear behaviour are assumed to develop at each incremented loading step, the non-linear load–displacement relationships can be approximately idealised as a series of tangential linear relationships corresponding to the multiple loading steps. This concept of linear approximation had already been applied to the non-linear analysis of the existing structures and members. The present study was focused on applying the well-known concept of incremental linear approximation to the inelastic seismic design of new structures.

Figure 4 shows the concept of the adaptive linear analysis. A

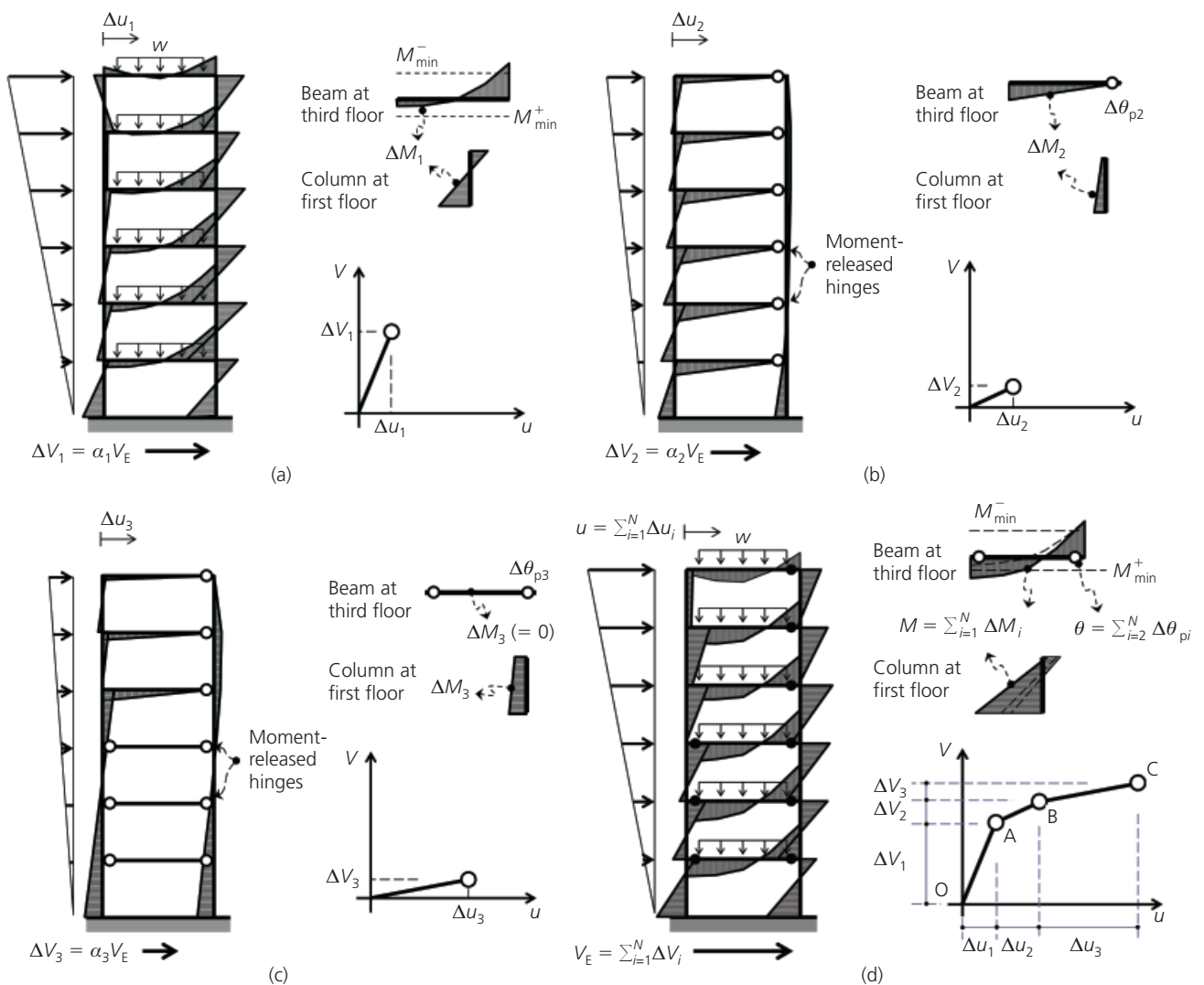


Figure 4. Concept of adaptive linear analysis for inelastic seismic design: (a) elastic behaviour ( $i = 1$ ); (b) inelastic behaviour ( $i = 2$ ); (c) inelastic behaviour ( $i = 3$ ); (d) final results ( $N = 3$ )

series of sequential linear analyses are performed separately for the multiple loading steps (Figures 4(a)–4(c)). After performing the linear analysis of a loading step, the analysis model of the structure is adapted and modified for the next loading step, addressing the current distribution of the plastic hinges. For example, when flexural yielding occurs at the left end of all beams at the first loading step ( $i = 1$ , see Figure 4(a)) the analysis model of the structure is modified by releasing or reducing rotational rigidities at the plastic hinges (see Figure 4(b)). After the sequential linear analyses for all loading steps, the design forces and the plastic deformations of the members are directly determined by simply summing up the results of all piecewise linear analyses (Figure 4(d)). The detailed procedures of the adaptive linear analysis for inelastic seismic design are as follows.

- (a) The minimum strength,  $M_{\min}$ , and the maximum plastic rotation,  $\theta_{pu}$ , of each member are specified. The elastic bending moments resulting from the elastic analysis for the gravity and wind load combinations can be used as  $M_{\min}$ . The minimum strength,  $M_{\min}$ , should not be less than the code requirements including the minimum reinforcement ratio (ACI 318-08 and KBC 2009). The maximum plastic rotation,  $\theta_{pu}$ , can be determined by using existing seismic guidelines such as FEMA-356 (ASCE, 2000) or experimental results, addressing performance objectives of the seismic design or/and ductility details of the members.
- (b) The number of loading steps,  $N$ , and the magnitude of lateral load increment in each loading step,  $\alpha_i E$ , are determined, where  $E$  denotes the design earthquake load and  $\alpha_i$  ( $0 < \alpha_i < 1$  and  $\sum_{i=1}^N \alpha_i = 1$ ,  $i = 1, \dots, N$ ) denotes the lateral load increment factor in a certain loading step,  $i$ . The values of  $N$  and  $\alpha_i$  which significantly affect the design results should be carefully chosen, as will be discussed later in this section.
- (c) Elastic behaviour: linear elastic analysis of the loading step  $i = 1$  (Figure 4(a)) using the load combination  $G + \alpha_1 E$  ( $G$  is the gravity load) is performed. The roof displacement increment,  $\Delta u_1$ , and the bending moment increment,  $\Delta M_1$ , of each member are then computed.
- (d) If the maximum value of the bending moment,  $\Delta M_1$ , in a beam is greater than the minimum strength,  $M_{\min}$ , it is supposed that a plastic hinge forms in that location. Therefore, the analysis model of the structure is modified for the next loading step  $i = 2$  by adding a moment-released hinge or rotational spring with post-yield stiffness,  $k_p$ , at the plastic hinge (see Figure 4(b)). When the gravity and earthquake loads are imposed simultaneously, the absolute value of the bending moment is higher at the beam end subjected to negative moment (Figure 4(a)). Therefore, in this step, the development of the plastic hinge is checked for the negative moment.
- (e) Inelastic behaviour: linear analysis of the subsequent loading step  $i$  ( $i = 2, \dots, N$ , see Figures 4(b) and 4(c)) for the lateral load increment,  $\alpha_i E$ , is carried out. The roof displacement

increment,  $\Delta u_i$ , the bending moment increment,  $\Delta M_i$  of each member, and the plastic rotation increment,  $\Delta \theta_{pi}$ , at each plastic hinge are then computed.

- (f) If the maximum value of the bending moment,  $\sum_{j=1}^i \Delta M_j$ , in a beam is greater than the minimum strength,  $M_{\min}$ , it is supposed that a new plastic hinge forms in that location. Therefore, the analysis model of the structure is modified for the next loading step ( $i + 1$ ) by installing an additional moment-released hinge or rotational spring with post-yield stiffness,  $k_p$ , at the new plastic hinge (see Figure 4(c)). When the plastic hinge has developed at the beam end subjected to negative moment in the previous loading steps, the development of the plastic hinge is checked for the positive moment. Otherwise, it should be checked for the negative moment. Only two plastic hinges are allowed in a beam: one at the location of the maximum negative moment and the other at the location of the maximum positive moment.
- (g) Linear analysis and the modification of the analysis model of the structure are repeated until  $i = N$ .
- (h) The final bending moment  $M$  of each member and plastic rotation,  $\theta_p$ , at each plastic hinge are calculated by the summation of all bending moment and plastic rotation increments (see Figure 4(d)).  $M = \sum_{i=1}^N \Delta M_i$  and  $\theta_p = \sum_{i=2}^N \Delta \theta_{pi}$ . Other design forces and deformations such as shear force, axial force, roof displacement and storey drift are also calculated from the linear analysis results for the multiple loading steps.
- (i) If the plastic rotations,  $\theta_p$ , are less than the maximum allowable plastic rotation,  $\theta_{pu}$ , the analysis results such as the bending moments,  $M$ , and plastic rotations,  $\theta_p$ , are acceptable for the inelastic seismic design. Otherwise, the adaptive linear analysis needs to be re-performed by modifying the load increment factor of each loading step,  $\alpha_i$ .
- (j) Reinforcements for the bending moments are determined at the critical sections where the maximum positive and negative moments occur. In addition, confinement reinforcements for the plastic rotations,  $\theta_p$ , are provided at the plastic hinges.

As shown in Figures 4(b) and 4(c), in the inelastic loading steps 2 through  $N$ , the moment redistribution occurs due to the development of the plastic hinges in the beams and the larger bending moments develop in the lower-storey columns due to the double cantilever action of the structure (see Figure 3(e)). Furthermore, the plastic deformations of the members are directly evaluated from the adaptive linear analysis.

The adaptive linear analysis results can vary depending on the lateral load increment factor,  $\alpha_i$ , which is determined from the engineer's judgement or the design conditions. For example, if an engineer uses 0.6, 0.9 and 1.0 times the design earthquake load for the structural performance levels required for immediate occupancy, life safety and collapse prevention, respectively, the linear analyses in three loading steps may be sequentially performed for  $\alpha_1 = 0.6$ ,  $\alpha_2 = 0.3$  and  $\alpha_3 = 0.1$ . In most cases,



the adaptive linear analysis using three loading steps ( $N = 3$ ) is sufficient to address the effects of the inelastic behaviour such as the moment redistribution and plastic rotations of members (see the next section on 'Application to inelastic seismic design' and subsection on 'Verification'). If more smooth analysis results are required, for example, in structures with irregularity or in high-rise buildings, a greater number of the loading steps  $N$  are used.

The distribution of the plastic hinges needs to be properly controlled to satisfy the storey drift limit state required for the performance objectives. For example, if an excessive storey drift is expected, the lateral load increment factor,  $\alpha_1$ , of the elastic behaviour is increased to reduce the inelastic deformation of the structure. To satisfy the strong column–weak beam concept, the plastic hinges are located only in the beams and the columns are controlled to remain elastic during the analysis process as shown in Figure 4. The development of plastic hinges at the first-storey columns is restrained until the final loading step ( $i = N$ ) in order to avoid premature failures of the columns (see Figure 4(d)).

If a conventional non-linear analysis is performed on the structure, where the moment strengths of the members are determined as the values resulting from the adaptive linear analysis, a load–displacement relationship that is exactly the same as that predicted by the proposed method will be obtained (O–A–B–C in Figure 4(d)). This indicates that the adaptive linear analysis for multiple loading steps can directly determine the forces and plastic deformations of the structures and members addressing the effects of the inelastic behaviour. However, this does not mean that the conventional non-linear analysis is unnecessary in the inelastic seismic design using the adaptive linear analysis. The adaptive linear analysis is aimed at the determination of inelastic forces and plastic rotations of the members in the preliminary seismic design where the non-linear analysis is not preferred. In practice, however, the design strength and ductility provided for the structure and the members can differ from those calculated by the analysis. Therefore, when accurate seismic evaluation is necessary, the conventional non-linear analysis needs to be used. This will be discussed in the next section.

The adaptive linear analysis can be applied to two- and three-dimensional regular and irregular structures. For practical use of the adaptive linear analysis in inelastic seismic design, a special computer program, which can automatically detect the formation of plastic hinges and can sum the results of sequential linear analyses, needs to be developed. The analysis procedures presented in this study are expected to be easily incorporated with existing analysis/design programs. However, as previously mentioned, the analysis results are sensitive to the lateral load increment factor, the number of loading steps and the distribution of plastic hinges. Therefore, further research is necessary to investigate in detail the effects of these variables on the forces and plastic deformations of the members

## Application to inelastic seismic design

### Design examples

The adaptive linear analysis for inelastic seismic design was applied to a six-storey moment frame. Figure 5 shows the configurations, gravity and earthquake loads, and dimensions of beams and columns. The moment frame has minor irregularities in the storey height, bay length and the size of beam sections. The storey heights are 4500 mm for the first storey and 3600 mm for the second to sixth storeys. The bay spans are 6000 mm for the exterior bays and 5000 mm for the interior bay. The dimensions of the cross sections are 450 mm  $\times$  650 mm and 450 mm  $\times$  750 mm for the beams, and 600 mm  $\times$  600 mm for the columns. The base shear is  $V_E = 1333$  kN. The vertical distribution of the earthquake load was determined according to KBC 2009 (AIK, 2009). The gravity load  $w_u = 90$  kN/m was applied to each beam. For elastic stiffness of members,  $0.7E_cI_g$  ( $= 2 \times 0.35E_cI_g$  including the effect of slabs) and  $0.7E_cI_g$  were used for the stiffness of the beams and columns, respectively, according to KBC 2009 and ACI 318-08.  $E_c$  is the elastic modulus of the concrete ( $= 25.7$  GPa), and  $I_g$  is the second-order moment of inertia of the gross section. The general purpose software, Midas-Gen (Midas-IT, 2005), was used for the adaptive linear analysis.

For the inelastic seismic design using the adaptive linear analysis, the following design conditions were considered.

- The moment frame is the special moment frame specified in ACI 318-08 and KBC 2009. To ensure the strong column–weak beam behaviour of the moment frame, the development of plastic hinges was allowed only to both ends of the beams. In principle, the plastic hinge can develop at the mid-span critical section of the beams where the bending moment is at a maximum. In this example, however, for simplicity the plastic hinge at the mid-span of the beams was not addressed.
- The maximum plastic rotation,  $\theta_{pu}$ , permitted in the beam plastic hinges was assumed as 0.03 rad (ASCE, 2000).
- When a plastic hinge develops in a beam, the flexural rigidity at the plastic hinge is supposed to be completely released ( $k_p = 0$ ).
- Elastic analysis for the gravity load,  $w_u$ , and wind load,  $W$  ( $V_W = 823$  kN, see Figure 5(a)), was performed and the bending moments resulting from the elastic analysis were used for the minimum strength,  $M_{min}$ , of each beam. When the moments resulting from the elastic analysis were less than the moment strength corresponding to the minimum reinforcement ratio of KBC 2009 and ACI 318-08, the latter was used for the minimum moment strength. The wind load was calculated from KBC 2009.
- The adaptive linear analysis was performed for three loading steps ( $N = 3$ ), whose load increment factors are  $\alpha_1 = 0.7$ ,  $\alpha_2 = 0.25$  and  $\alpha_3 = 0.05$ , respectively. In the linear analysis of each load increment, the second-order effect was addressed.

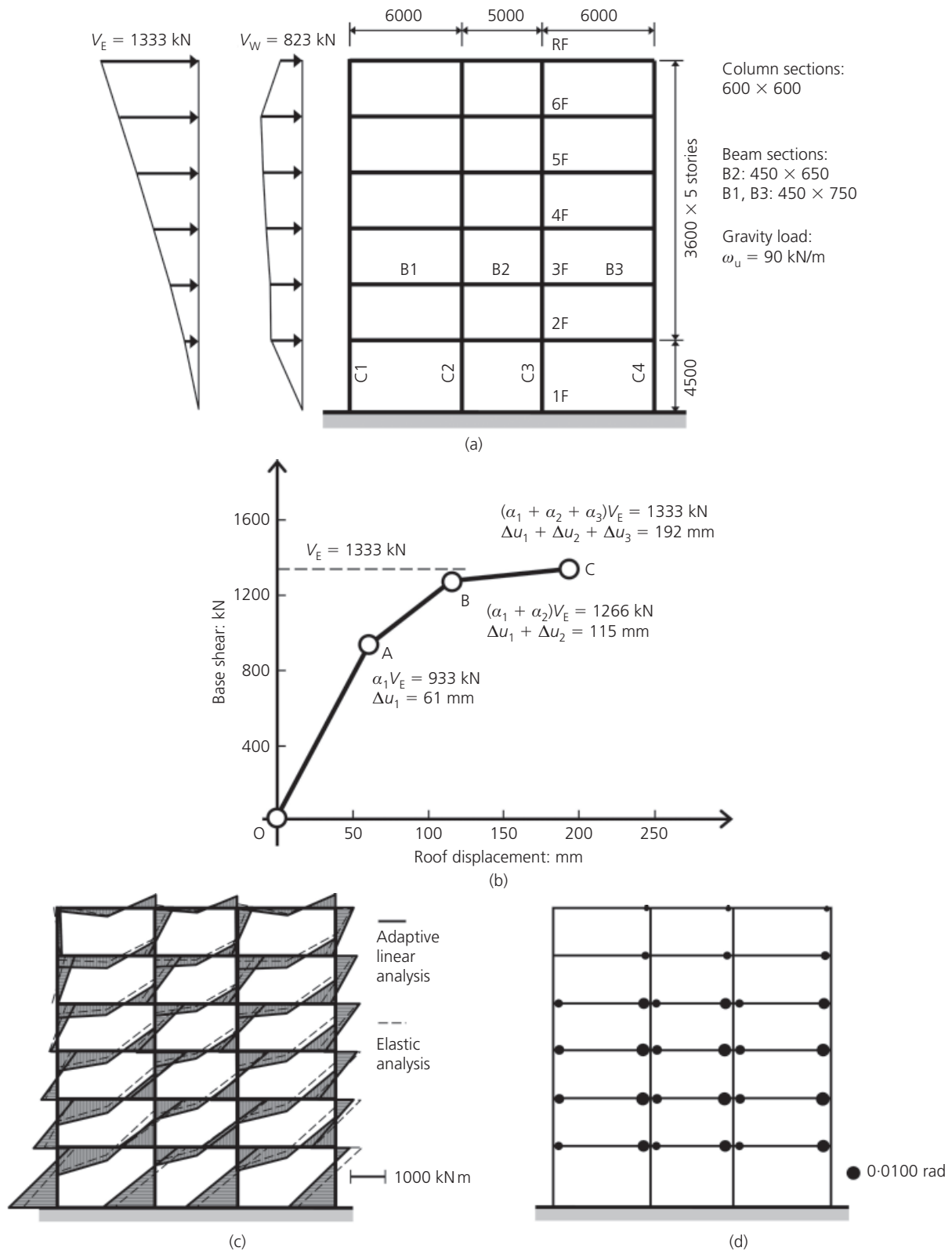


Figure 5. Results of adaptive linear analysis of six-storey regular moment frame: (a) configurations (mm) and design loads; (b) base shear–roof displacement relationship ( $N = 3$ ); (c) design moments ( $N = 3$ , kN m); (d) member plastic rotations ( $N = 3$ , rad)



The adaptive linear analysis for the six-storey regular moment frame ( $N = 3$ ) was carried out to determine bending moment and plastic rotation demands on the members addressing the effects of inelastic behaviour. Figure 5(b) shows the base shear–roof displacement relationship obtained by superposing the piecewise linear analysis results of all loading steps. The load-carrying capacity of the frame was identical to the design base shear ( $V_E = 1333$  kN) and thus the over-strength associated with the effects of the inelastic behaviour, such as the moment redistribution between the members and the plastic mechanism of the structure, was diminished. The bending moments of the members determined from the adaptive linear analysis are shown in Figure 5(c). For comparison, the bending moments by elastic analysis are also shown in the figure. Owing to the moment redistribution, the bending moments of the first-storey columns were significantly increased. In addition, the positive bending moments of the beams were increased while the negative bending moments were decreased. ACI 318-08 and KBC 2009 require that in the special moment frame, the positive moment strength at a beam critical section near the beam–column joint be greater than one-half of the negative moment strength in order to assure the ductile behaviour of the beam. As shown in Figure 5(c), the bending moments predicted by the adaptive linear analysis satisfied the requirement of the special moment frame well, owing to the moment redistribution, while those predicted by the elastic analysis did not satisfy the requirement. The plastic rotations of the beams calculated by the adaptive linear analysis are also shown in Figure 5(d). None of the beam plastic rotations exceeded the maximum allowable plastic rotation  $\theta_{pu} = 0.03$  rad at a roof displacement of 192 mm (C in Figure 5(b)). The plastic hinges were evenly distributed at all storeys. These aspects indicate that the proposed method can secure a robust ductile behaviour by taking the inelastic effects into account in the preliminary design.

The adaptive linear analysis was also applied to the inelastic seismic design of a nine-storey special moment frame with vertical irregularity. As shown in Figure 6(a), the moment frame has set backs in the lower storeys. The storey heights are 4500 mm for the first to third storeys and 3600 mm for the fourth to ninth storeys, and the bay span is 6000 mm. The dimensions of the cross sections are 450 mm  $\times$  750 mm and 450 mm  $\times$  650 mm for beams and 800 mm  $\times$  800 mm and 700 mm  $\times$  700 mm for columns. The base shear is  $V_E = 1484$  kN. The gravity loads  $w_u = 90$  kN/m and 80 kN/m were applied to the beams at the second to fourth floors and at the fifth to roof floors, respectively. The design conditions used for the regular moment frame shown in Figure 5 were also used for the irregular moment frame.

Figure 6(b) shows the base shear–roof displacement relationship obtained by superposing the piecewise linear analysis results of three loading steps. The bending moments of the members determined from the adaptive linear analysis and elastic analysis are shown in Figure 6(c). The plastic rotations of the beams calculated by the adaptive linear analysis are also shown in

Figure 6(d). As shown in Figure 6, the aspects of the adaptive linear analysis in the irregular moment frame were very similar to those in the regular moment frame shown in Figure 5. The positive bending moments of the beams were increased while the negative bending moments were decreased because of the moment redistribution; the plastic hinges were evenly distributed at all storeys.

### Verification of design results

The regular and irregular moment frames shown in Figures 5 and 6, respectively, designed by the adaptive linear analysis were examined by the non-linear analysis using Drain-2DX (Prakash *et al.*, 1993). For frame models for the non-linear analysis, it was assumed that plastic hinges can develop only at both ends of the beams and columns and an elastic-perfectly plastic behaviour was used for the plastic hinges ( $k_p = 0$ ) (see design conditions (a) and (c) of the ‘Design examples’). The nominal yield strengths of the plastic hinges were determined as follows.

The bending moments calculated by the adaptive linear analysis were used as the nominal yield strength,  $M_y$ , at the ends of the beams and columns. When the bending moment of a beam or a column determined from the adaptive linear analysis was less than the given minimum strength,  $M_{min}$ , of the beam or the column (see design condition (d) in ‘Design examples’),  $M_{min}$  was used as the nominal yield strength,  $M_y$ , of the beam or the column. Two requirements on the special moment frame specified in ACI 318-08 and KBC 2009 were considered: When the sum of the nominal yield strengths of the columns at a beam–column joint was less than 1.2 times that of the beams framing into the joint, the nominal yield strengths of the columns were increased so as to satisfy the strong column requirement; when the positive yield strength at a beam end was less than one-half of the negative yield strength, one-half of the negative yield strength was used as the positive yield strength of the beam.

A displacement-controlled pushover analysis was carried out for the six-storey regular and nine-storey irregular moment frames shown in Figures 5 and 6, respectively. The base shear–roof displacement relationships (thin solid lines with diamonds) predicted by Drain-2DX are shown in Figures 7(a) and 8(a). The non-linear analysis was stopped at points D’ where the maximum storey drift ratio reaches  $\delta_u = 0.025$ .  $\delta_u = 0.025$  is the allowable maximum inelastic storey drift ratio of the special moment frame (AIK, 2009). As shown in the figures, the load-carrying capacities of the moment frames were degraded owing to the second-order effect as the roof displacement was increased. The adaptive linear analysis showed greater load-carrying capacities than those of the Drain-2DX analysis, even though the second-order effect was addressed. This is because the adaptive linear analysis performed a number of segmental linear analyses, and thus elastic and inelastic lateral displacements resulted from the previous loading steps did not contribute to the second-order effect of the next loading step.

Figures 7(b) and 8(b) show the member plastic rotations of the

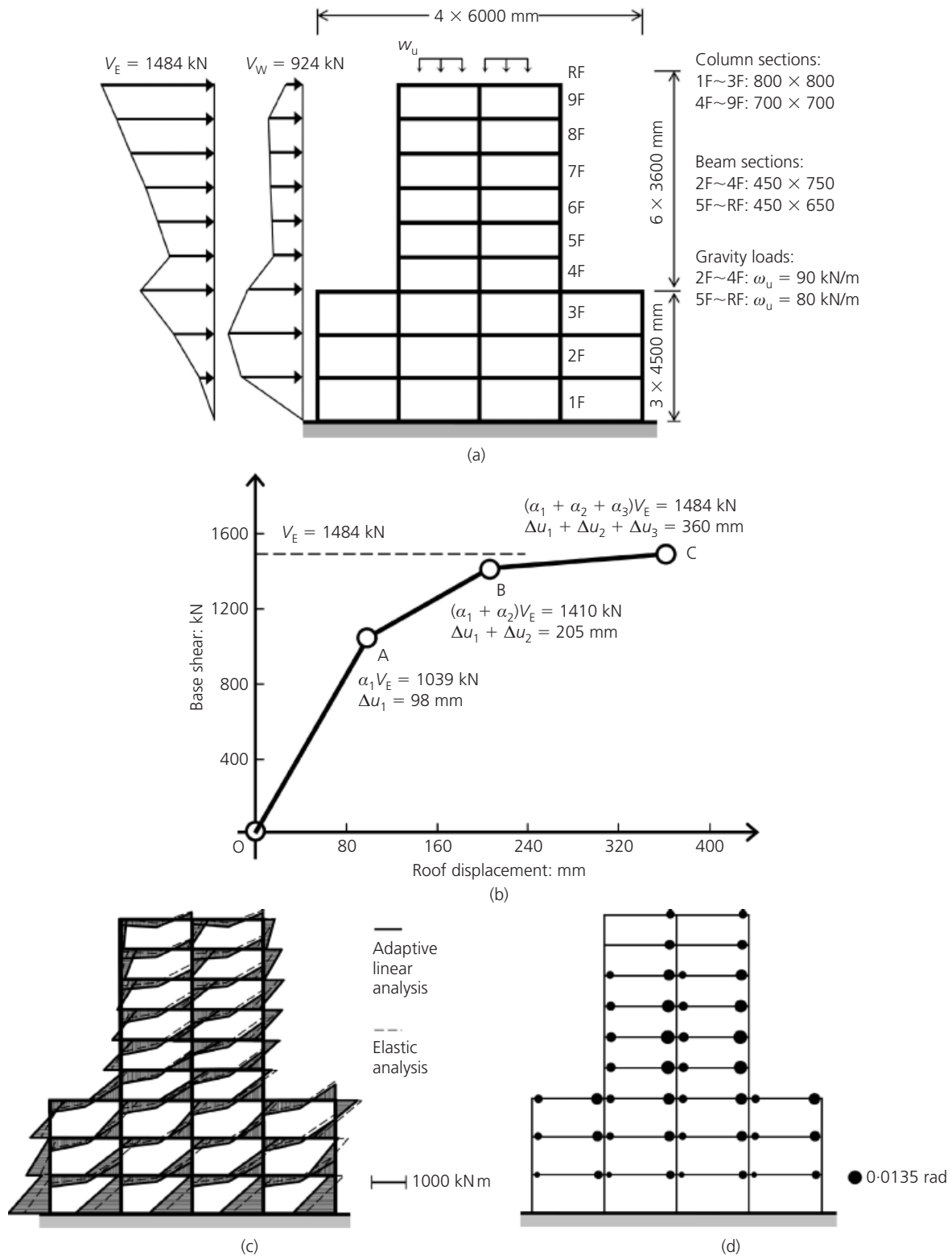
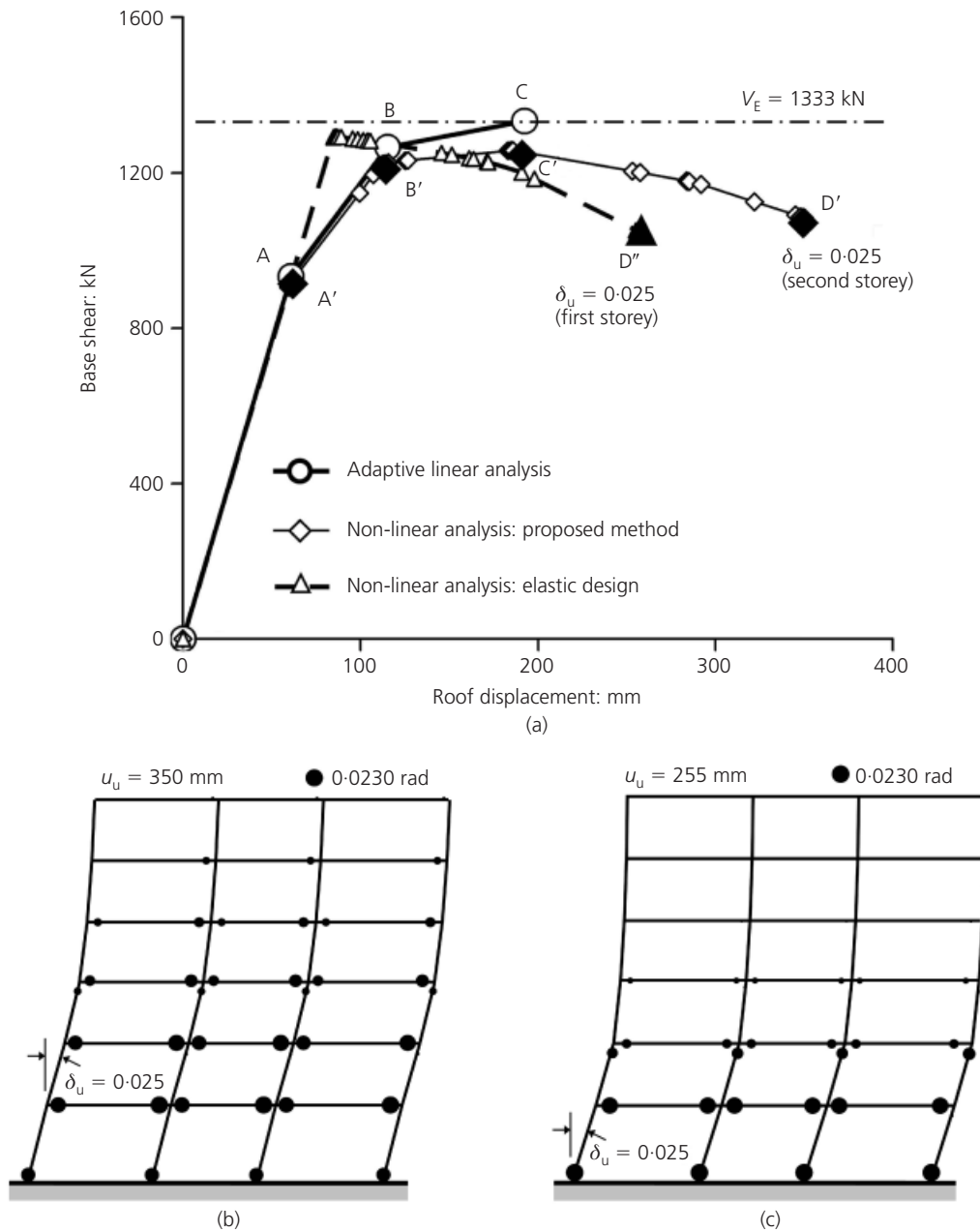


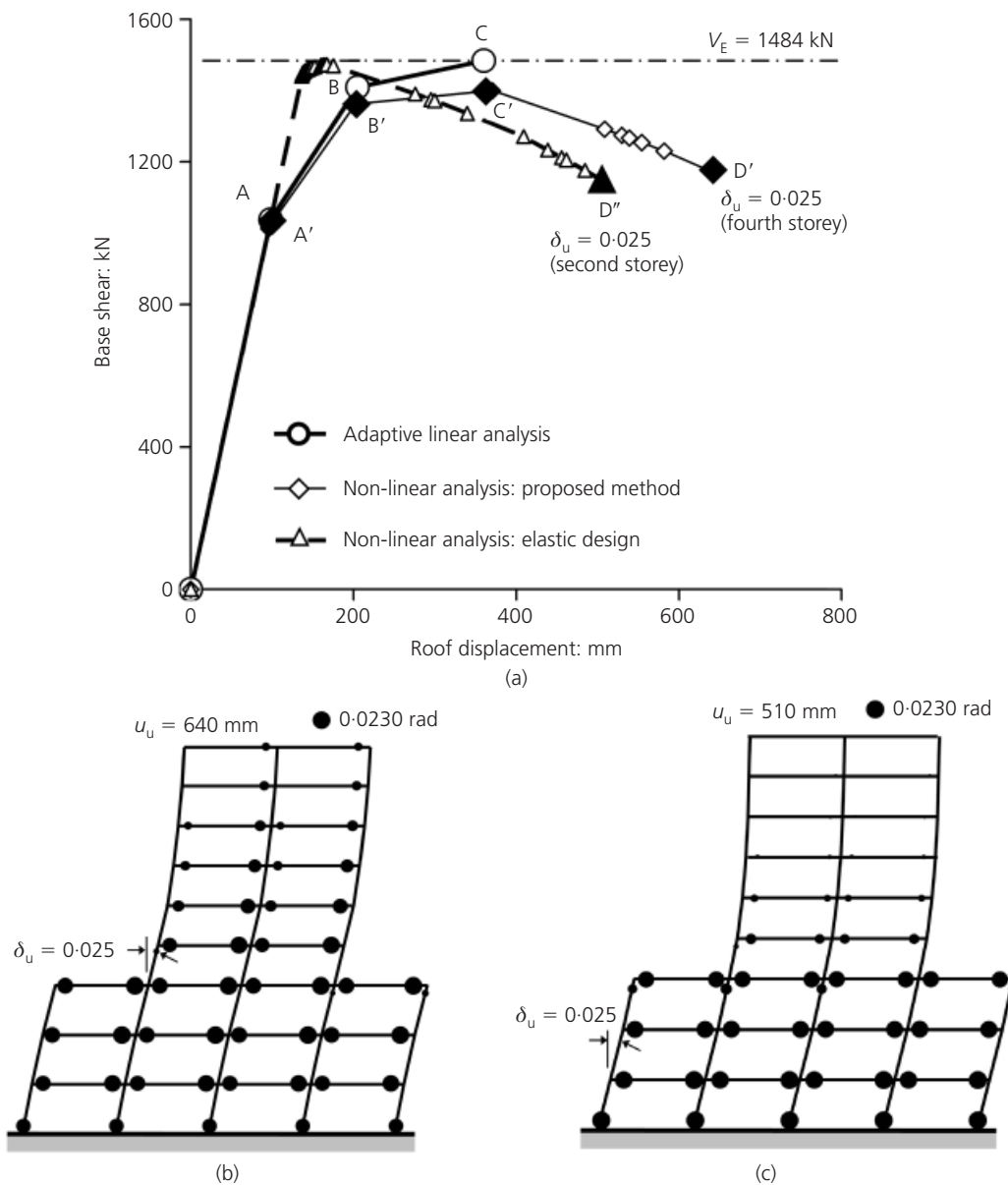
Figure 6. Results of adaptive linear analysis of nine-storey irregular moment frame: (a) configurations (mm) and design loads; (b) base shear–roof displacement relationship ( $N = 3$ ); (c) design moments ( $N = 3, \text{ kN m}$ ); (d) member plastic rotations ( $N = 3, \text{ rad}$ )



**Figure 7.** Non-linear analysis of six-storey regular moment frame designed by the proposed method and elastic analysis: (a) base shear–roof displacement relationship; (b) deformed shape and plastic rotations at D' (proposed method); (c) deformed shape and plastic rotations at D'' (elastic design)

six-storey regular and nine-storey irregular moment frames at points D', respectively. At points C', plastic hinges were developed only at beams as predicted by the adaptive linear analysis (see Figures 5(d) and 6(d)). However, as the roof displacement was increased further beyond points C', yielding in columns was observed at the bottom of the first storey and at the top of the middle and upper storeys. However, at points D' where the

maximum storey drift ratio is  $\delta_u = 0.025$ , plastic rotations were evenly developed at all beams and the columns of the first storey, while the plastic rotations were not significant at the columns of the second to roof storeys. This indicates that the inelastic seismic design using the adaptive linear analysis can successfully assure the strong column–weak beam design concept and thus a greater ductility of the moment frames can be achieved.



**Figure 8.** Non-linear analysis of nine-storey irregular moment frame with vertical irregularity designed by the proposed method (a) base shear–roof displacement relationship; (b) deformed shape and plastic rotations at D' (proposed method); (c) deformed shape and plastic rotations at D'' (elastic design)

### Comparison with elastic analysis

The inelastic deformations such as roof displacements, storey drifts and plastic rotations of beams predicted by the adaptive linear analysis differ from the inelastic deformation demands in seismic design, which correspond to the storey drift ratio of  $\delta_u = 0.025$  (compare the points C' and D' in Figures 7(a) and 8(a)). In principle, the inelastic deformations predicted by the adaptive linear analysis (e.g. points C' in Figures 7(a) and 8(a)) are consistent with the deformations when the bending moments

have been redistributed between the members and the plastic mechanism of the structure starts to develop. However, the actual deformation demands in the seismic design (e.g. points D' in Figures 7(a) and 8(a)) may be greater or less than the inelastic deformations predicted by the adaptive linear analysis, depending on the earthquake intensity and target ductility. However, even when the inelastic deformations predicted by the adaptive linear analysis are less than the deformation demands in the seismic design, the adaptive linear analysis can achieve a better ductility

of the structure than the elastic analysis. This is clear when the results of the non-linear analysis on the frames designed by both analysis methods are compared, as follows.

The moment frames in Figures 5(a) and 6(a) re-designed by the elastic analysis were analysed by Drain-2DX. The frame models for the non-linear analysis and the plastic hinge behaviour were exactly the same as those used in the previous subsection 'Verification of design results'. However, the nominal yield strengths  $M_y$  at both ends of the beams and columns were determined as the bending moments predicted by the elastic analysis (see dashed lines in Figures 5(c) and 6(c)). When determining the nominal moment strength at the plastic hinges, code requirements on the minimum moment strength such as the minimum reinforcement ratio, the column-to-beam moment strength ratio at beam-column joints, and the positive-to-negative moment strength ratio at beams, which are specified in ACI 318-08 and KBC 2009, were used (see subsection on 'Verification of design results'). The non-linear analysis was performed until points D' where the maximum storey drift ratio reaches  $\delta_u = 0.025$ .

Figures 7(a) and 8(a) show the base shear-roof displacement relationships of the six-storey regular and nine-storey irregular moment frames, respectively, designed by the elastic analysis (dashed lines with triangles). For the frames designed by the elastic analysis, the yielding of beams and columns was delayed until the load-carrying capacities reached the design earthquake load  $V_E$ . However, the softening during the post-yield behaviours was more significant than that of the frames designed by the adaptive linear analysis (thin solid lines with diamonds). In addition, the maximum roof displacements at points D' where the maximum storey drift ratio reaches  $\delta_u = 0.025$ , were much less than those of the adaptive linear analysis, and thus the frames designed by the elastic analysis have less ductility.

Figures 7(c) and 8(c) show the deformed shapes of the frames and the plastic rotations of beams and columns at points D'. For the frames designed by the elastic analysis, the storey drifts and plastic rotations were concentrated on lower storeys, while for the frames designed by the adaptive linear analysis the storey drifts and plastic rotations were distributed evenly over all storeys and members (compare Figures 7(b) and 7(c) and Figures 8(b) and 8(c)). The columns designed by the elastic analysis showed more significant plastic rotations at their tops or bottoms. Even a soft storey mechanism was observed in the six-storey regular moment frame (see Figure 7(c)). These observations indicate that the elastic analysis and the column-to-beam moment strength ratio greater than 1.2, which are used for the special moment frames in current design codes, including ACI 318-08 and KBC 2009, may not be sufficient to assure a strong column-weak beam behaviour with a good ductility.

## Conclusions

Structures subjected to strong earthquakes generally experience inelastic deformation. Therefore, to ensure seismic safety, the

effects of the inelastic behaviour such as the moment redistribution and plastic deformation need to be considered in the design process. In the present study, non-linear analysis was performed to investigate the lateral load transfer mechanism of the moment frame. Based on the analysis results, a sequential linear analysis, the adaptive linear analysis, was developed based on the concept of incremental linear approximation for the non-linear analysis. In the adaptive linear analysis, a series of linear analyses are performed separately for multiple earthquake loading steps. After performing the linear analysis for each load increment, the analysis model of the structure is modified according to the current plastic hinge distribution for the linear analysis of the next loading step. Then, by simply summing up the results of all linear analysis results, the inelastic member forces and plastic deformations are directly determined.

The adaptive linear analysis was applied to the inelastic seismic design of regular and irregular special moment frames specified in ACI 318-08 (ACI, 2008) and KBC 2009 (AIK, 2009). The design results of the moment frames were verified by non-linear analysis. Unlike elastic analysis, the design moments of the beams and columns were determined addressing the moment redistribution. In addition, the plastic rotations of the members were directly evaluated. Since the plastic mechanism and moment redistribution are directly addressed in the analysis, the over-strength was minimised and the unexpected plastic mechanisms were avoided.

The adaptive linear analysis can address the effects of inelastic behaviour (e.g. plastic mechanism and moment redistribution) in preliminary seismic design, without complicated non-linear analysis. This ensures that the inelastic seismic design of structures is more easily accessible to structural engineers. However, this does not mean that conventional non-linear analysis is unnecessary for the inelastic seismic design using the adaptive linear analysis. The adaptive linear analysis method is aimed at the application to the preliminary design, and if accurate seismic evaluation is necessary, the sophisticated non-linear analysis should be performed.

The philosophy of performance-based seismic design can be easily implemented using the adaptive linear analysis since the desired plastic mechanisms and specific seismic design strategies, such as the strong column-weak beam concept, can be directly implemented in the analysis process. However, the results of the adaptive linear analysis can be significantly affected by the lateral load increment factor, the number of loading steps and the distribution of plastic hinges. Therefore, these parameters need to be carefully chosen in inelastic seismic design.

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