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Optimum distribution of added viscoelastic dampers for mitigation of torsional responses of plan-wise asymmetric structures

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Abstract

In this study a strategy was developed for an appropriate plan-wise distribution of viscoelastic dampers to minimize the torsional responses of an asymmetric structure, with one axis of symmetry subjected to an earthquake-induced dynamic motion. The modal characteristic equations of a single-storey asymmetric structure with four corner columns and added viscoelastic dampers were derived, and a parametric study was performed to identify the design variables that influence the torsional responses. Based on the results of parametric study, a simple and straightforward methodology to find out the optimum eccentricity of added VED to compensate for the torsional effect of a plan-wise asymmetric structure was developed using modal coefficients. The results indicate that the torsional response of asymmetric structures can be reduced significantly following the proposed method, and that the viscoelastic dampers turn out to be more effective than viscous dampers in controlling torsional response of a plan-wise asymmetric building structure.

Keywords: Plan-asymmetry; Stiffness eccentricity; Optimum damper eccentricity; Viscoelastic dampers

1. Introduction

The structural irregularities such as irregular distribution of mass, stiffness, or strength in their floor plan may lead to damages much enlarged compared to the case of structures with their properties symmetrically distributed. Previous experiences indicate that those structures with irregularities are especially vulnerable to earthquake-induced dynamic motions.

Research related to reducing torsional effect of plan-wise asymmetric structure first started from understanding elastic behavior of a structure arising from asymmetry [1]. Later the research focused on investigating the inelastic response of asymmetric structures to give design guidelines [2–4]. More recently, another research effort has been imparted on the reduction of torsional responses of a single-storey structure by use of supplemental viscous dampers (VD). Goel [5] showed that the torsional response of an asymmetric single-storey structure can be reduced by locating VD asymmetrically. He identified three parameters that control the influence of VD on the linear response of an asymmetric structure. He also investigated the role of VD in the change of modal characteristics of the asymmetric structure, and studied the dynamic response to harmonic ground motion [6]. He found that most modal parameters, except the dynamic amplification factor, are affected little by the plan-wise distribution of supplemental damping, and that the trends for modal deformations are directly related to the apparent modal damping ratios. Lin and Chopra [7] extended Goel's study, and tried to improve the understanding of how and why plan-wise distribution of dampers influences the response of one-storey asymmetric structure.

From a practical point of view, however, further research is still required regarding how to distribute added dampers on a plan to mitigate the torsional responses effectively. Moreover, it can be expected that the viscoelastic dampers (VED), which provides stiffness as well as viscous damping, can be more effective than the viscous dampers in the mitigation of torsional responses.

The objectives of this study are: (1) to investigate how

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the dynamic characteristics and elastic response of torsionally-coupled structures is influenced by the plan-wise distribution of VED; and (2) to propose a simple and straightforward methodology to find out the optimum eccentricity of added VED to compensate for the torsional effect of a plan-wise asymmetric structure with one axis of symmetry. To accomplish these objectives the dynamic characteristic equation of a single-storey structure with four corner columns, symmetric along one axis but asymmetric along the perpendicular axis, was derived. Using the characteristic equation a diagram for optimum damper locations was constructed as a design guide, which provides the optimum damper eccentricity for the given parameters such as stiffness eccentricity, damping ratio, and frequency ratio.

It should be pointed out, however, that the main reason of installing supplemental dampers is to mitigate the overall dynamic responses, not to reduce the torsional responses. To reduce torsional responses it would be more effective to place structural members, such as steel braces or concrete walls, to balance the stiffness. Therefore the idea of the present study is to maximize the usability of VED when they are required for reduction of overall dynamic responses of a plan-wise asymmetric structure.

2. Characteristics of viscoelastic dampers

A typical viscoelastic damper (VED) consists of thin layers of viscoelastic material bonded between steel plates. The dynamic behavior of VED is generally represented by a spring and a dashpot connected in parallel as shown in Fig. 1. Although more accurate methods of analytical modeling exist, such as based on Boltzmann's superposition principle [9] or on fractional derivative constitutive relationship [10], they may not be applicable in practice for their huge computational demands. For the linear spring-dashpot representation of the damper, the stiffness k_d and the damping coefficient c_d are obtained as follows [11]:

$$k_d = \frac{G'(\bar{\omega})A}{t} \qquad c_d = \frac{G''(\bar{\omega})A}{\bar{\omega}t} \tag{1a, b}$$

where $G'(\bar{\omega})$ and $G''(\bar{\omega})$ are the shear storage and shear loss moduli, respectively, and A and t are the total shear area and the thickness of the material, respectively, and $\bar{\omega}$ is the forcing frequency, for which the fundamental natural frequency of the structure is generally utilized in time domain analysis. In this study the ratio of shear storage and shear loss moduli will be denoted as γ, i.e. $\gamma = \frac{G'}{G''}$. With this spring-damper idealization the dynamic system matrices of the structure with added VED can be obtained by superposing the damper properties to the stiffness and damping matrices of the structure.

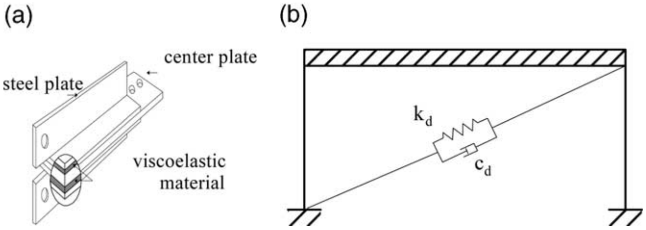

Fig. 1. Typical shape and mathematical modeling of a viscoelastic damper.

3. Dynamic characteristics of a plan-wise asymmetric structure

Structures subjected to an earthquake excitation undergo torsional as well as lateral motions if their mass and stiffness centers do not coincide. As a result of coupled lateral–torsional motions, the lateral displacements and member forces may increase significantly compared to those experienced by the same structure with symmetric plan. In this study the dynamic equations of motion of a single-storey asymmetric structure with four corner columns, installed with added viscoelastic dampers, were derived. Then parametric studies were carried out to investigate the effects of design variables on the modal characteristics of the structure.

3.1. Model structure

The model structure to be investigated in this study is the idealized single-storey structure consisting of four corner columns and a rigid floor as shown in Fig. 2. The floor mass m is uniformly distributed over the plan, and consequently the center of mass (CM) is identical to the center of geometry. The center of supplemental damper (CSD) is defined the same way as the center of rigidity (CR) is generally defined. The stiffness is symmetric about the X-axis, while it is not symmetric about the Y-axis. Therefore the structure will undergo coupled lateral–torsional motions for earthquake load acting along the Y-axis. The normalized supplemental damping eccentricity, $\bar{e}_{sd} = \frac{e_{sd}}{a}$, is defined as the distance between the CM and the CSD divided by the dimension of the floor, a. Similarly the distance between the CM and the CR divided by a is defined as the stiffness eccentricity, $\bar{e} = \frac{e}{a}$. The responses to be considered are the peak displacements at the flexible and the stiff edges of the asymmetric structure. The relative displacements of both edges compared to those of the corresponding symmetric-plan structure will indicate the effects of plan

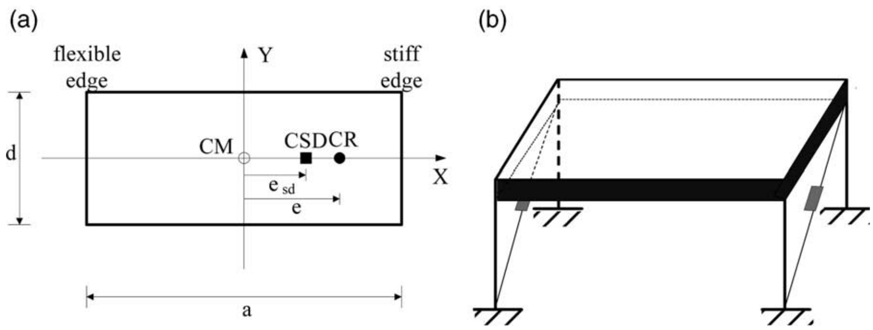

Fig. 2. One-storey asymmetric-plan system with supplemental viscoelastic dampers.

asymmetry. It can be expected that while a plan-wise uniform distribution of the supplemental dampers may lead to a reduction in the overall response of the system, displacement of the flexible edge will be reduced most effectively by distributing the VED in such a way that the center of supplemental dampers is located on the opposite side of the center of mass from the center of rigidity, as was already shown by Goel [5] for VD.

3.2. System matrices of the model structure with added dampers

The model structure has two DOF's when subjected to a ground motion along the Y-axis: translation along the Y-axis and rotation about a vertical axis. The uncoupled displacement along the X-axis will be neglected for simplicity. Then the displacement vector $\mathbf{u}$ for the system can be defined as $\mathbf{u}^T = \{u_y, \; au_\theta\}$ where u_y is the lateral relative displacement of the CM along the Y-axis, u_θ is the rotation of the floor about a vertical axis, and a is the plan dimension of the system along the X-axis. The mass matrix of the model structure and the damping matrix contributed from added dampers can be obtained as follows [6]:

$$\mathbf{M} = m\begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1+\alpha^2}{12\alpha^2} \end{bmatrix} \qquad \text{(2a, b)}$$

$$\mathbf{C} = 2m\omega_y\xi_{sd}\begin{bmatrix} 1 & \bar{e}_{sd} \\ \bar{e}_{sd} & \bar{e}_{sd}^2+\rho^2 \end{bmatrix}$$

where m is the mass of the structure concentrated in the floor level, $\alpha = a/d$ is the aspect ratio of the plan, ω_y is the lateral vibration frequency of the corresponding uncoupled system, ξ_{sd} is the damping ratio of the system due to supplemental dampers, and ρ is the normalized supplemental damping radius of gyration, which is the distance between the center of supplemental dampers and the damper location divided by the width of the plan, a. According to Goel [6], the effect of the supplemental dampers is maximized when the dampers are located as far as possible from the center of mass. Therefore throughout this study the dampers will be located at the edges, and ρ has the fixed value of 0.5.

The mass and the damping matrices of the model structure with added VED, Eq. (2), are actually the same with those obtained for asymmetric structure with added VD. However the system stiffness matrix becomes different due to the inherent stiffness of VED, which can be obtained as follows (see the Appendix for the derivation):

$$\mathbf{K} = \frac{m\omega_y^2}{\mu}\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix}; \; k_{11} = \frac{m\omega_y^2}{\mu}, \; k_{12} = m\omega_y^2\Big(\bar{e} + \bar{e}_{sd}\frac{2\xi_{sym}\gamma}{\mu}\Big), \; k_{21} = m\omega_y^2\Big(\bar{e} + \bar{e}_{sd}\frac{2\xi_{sym}\gamma}{\mu}\Big), \; k_{22} = \frac{m\omega_y^2}{\mu}\times\left[\Big(\big(\bar{e} + \bar{e}_{sd}\frac{2\xi_{sym}\gamma}{\mu}\big)\mu\Big)^2 + \frac{(\lambda^2-2\lambda^2\xi_{sym}\gamma-1)\,\Omega^2 + 6\lambda^2\xi_{sym}\gamma}{12\gamma^2-4\Omega^2}\right] \qquad (3)$$

where, $\mu = \dfrac{1}{1-2\xi_{sym}\gamma}$, $\lambda = \omega_x/\omega_y$, $\gamma = \dfrac{G'}{G''}$, $\Omega = \dfrac{\omega_\theta}{\omega_y}$, and ξ_{sym} is the damping ratio of the corresponding symmetric system.

Once the velocity-dependent supplemental dampers, such as VD and VED, are added, the structure generally becomes a nonproportional damping system, in which the damping matrix (Eq. (2) (b)) is not proportional to mass or stiffness matrix of the system. Therefore in this study the eigenvalue problem was solved in complex domain, and only real parts of components of mode shape vectors and absolute values of modal participation factors were utilized.

4. Parametric study

Using the characteristic equations derived in the previous section, parametric studies were performed to investigate the variation of the dynamic characteristics and responses of the asymmetric model structure for the change in design variables such as damping eccentricity ($\bar{e}_{sd}$), damping ratio of the corresponding symmetric system (ξ_{sym}), and the ratio of storage and loss moduli (γ). The modal characteristics considered are natural frequencies ($\omega_{1,2}$) and modal damping ratios ($\xi_{1,2}$). The stiffness eccentricity $\bar{e}$ was varied from 0 to 0.3, the damping eccentricity $\bar{e}_{sd}$ from 0 to -0.3 (opposite direction to the stiffness eccentricity), the damping ratio of the fundamental mode in a corresponding symmetric structure ξ_{sym} from 3% (inherent damping ratio of the structure) to 30%, and γ from 0 to 1. Here γ=0 represents the case of VD, and thus it is possible to compare the effect of both the VD and VED by controlling γ.

4.1. Variation of natural frequencies

Fig. 3 describes the change in the first and the second natural frequencies for various parameters. It can be seen that the natural frequencies are not affected significantly by the change in $\bar{e}_{sd}$ and $\bar{e}$. However as γ and the damping ratio increase, the natural frequencies also increase, due to the stiffness of the added dampers.

4.2. Variation of modal damping ratios

Fig. 4 shows that for ξ_{sym}=30% the damping ratio for the fundamental mode becomes maximum when the dampers are located in such a way that the damping eccentricity $\bar{e}_{sd}$ is about -0.2. Likewise it can be verified that the damping ratio is maximized at around $\bar{e}_{sd}$=-0.35 when ξ_{sym}=20%. This demonstrates that, for a given amount of dampers, there exists an optimum damper eccentricity that maximizes the modal damping ratio. It can be observed in Fig. 4 (b) that as γ becomes 0, i.e. as the stiffness of VED goes to 0, the damping ratio of the fundamental mode, ξ_1, increases.

4.3. Displacements at the mass center

Fig. 5 shows the variation of the maximum displacements at the mass center of the model structure with ξ_{sym}=30% subjected to the El Centro (NS) earthquake, which is known to represent one of the most severe combinations of strong motion ground acceleration over a long duration [4]. The lateral displacements at CM contributed from the first and the second vibrational modes, u_{d1} and u_{d2}, are plotted in Fig. 5 (a) and (b), respectively, for various stiffness and damping eccentricities. When the system is symmetric, i.e. $\bar{e} = 0$, the modal displacement u_{d1} increases monotonically as $\bar{e}_{sd}$ increases. When a stiffness eccentricity exists, there can be found a certain damper eccentricity that minimizes the modal displacement caused by the first mode vibration. The same phenomenon can be observed in Fig. 5 (c) in the variation of the rotation contributed from the first mode vibration. Contrary to the responses associated with the first mode, those contributed from the second mode increase monotonically as the damping eccentricity increases (Fig. 5 (b) and (d)). However, considering that the modal participation of the fundamental mode is dominant in most cases, it can be expected that the overall response of a plan-wise asymmetric structure can be diminished by proper asymmetric distribution of supplemental dampers. It is interesting to note that the damper eccentricities that minimize the lateral and rotational responses are almost identical. Fig. 5 (e) and (f) present the variation of total lateral and rotational displacements as a function of γ. It can be observed that the total displacement decreases as γ increases, i.e. as the stiffness of the damper increases.

4.4. Displacements at the stiff and flexible edges

Figs. 6 and 7 represent the displacements at each edge for El Centro and Taft earthquakes, respectively. When the dampers are symmetrically distributed, i.e. $\bar{e}_{sd}$=0, the displacement at the stiff edge decreases and that of the flexible edge increases as the stiffness eccentricity $\bar{e}$ increases (Figs. 6 and 7 (a)). This is none other than the general characteristics of a plan-wise asymmetric structure. Figs. 6 and 7 (b) depict the edge displacements for $\bar{e}$=0.2 as a function of damper eccentricity. It can be observed that when the dampers are located symmetri-

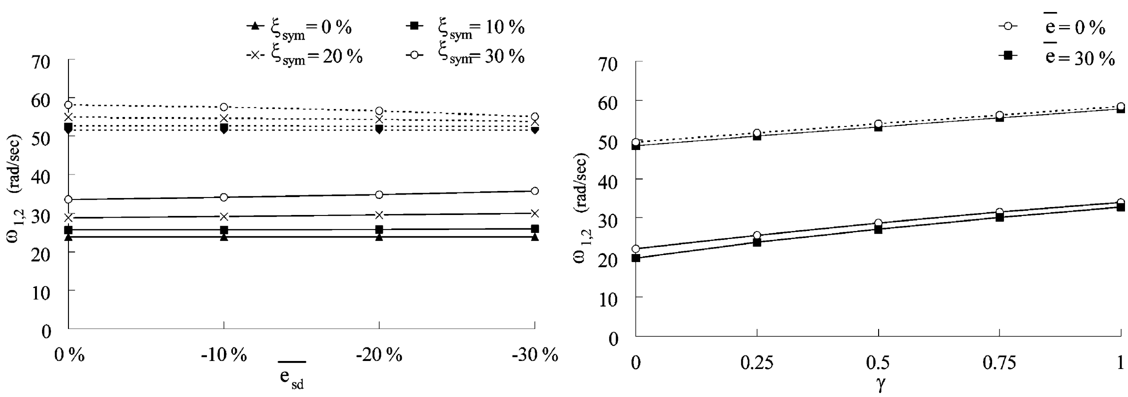

Fig. 3. Change in natural frequencies ((a) $\gamma = 1$, $\bar{e} = 0.2$; (b) $\xi_{sym} = 0.3$, $\bar{e}_{sd} = 0$).

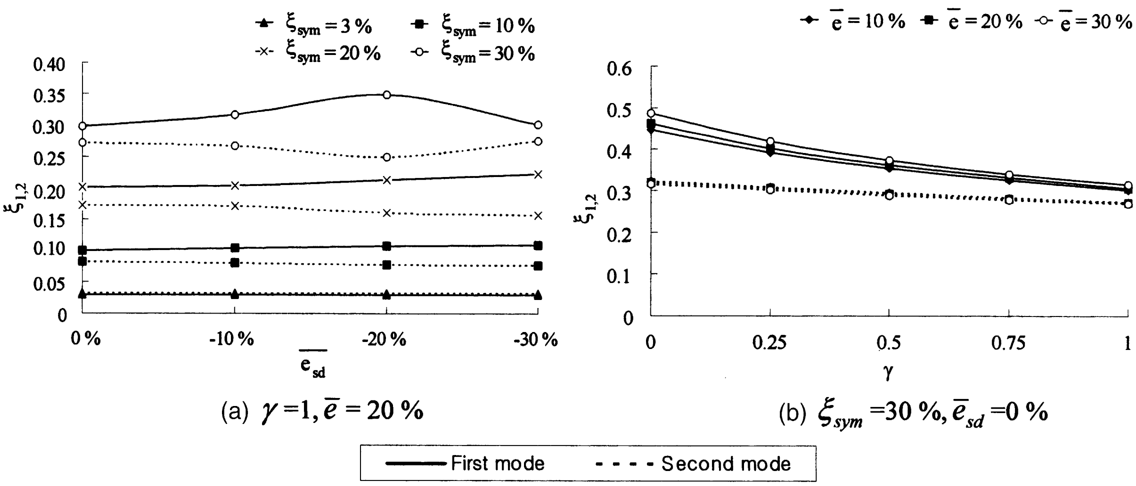

Fig. 4. Change in modal damping ratios ((a) $\gamma = 1$, $\bar{e} = 0.2$; (b) $\xi_{sym} = 0.3$, $\bar{e}_{sd} = 0$).

cally, the displacements at the stiff edge are smaller than those at the flexible edge regardless of the damping ratio. However, as the damper eccentricity increases, the displacements at stiff edge increases and those at flexible edge decreases until the displacements of the two edges become identical, which is considered to be the point of optimum damper eccentricity. It also can be noticed that the optimum damper eccentricity decreases as the damping ratio increases, i.e. as the amount of the supplemental dampers increases. Finally Figs. 6 and 7 (c) show that the displacements at both edges decrease monotonically as γ increases, i.e. as the stiffness of the dampers increases. This observation, together with the results shown in Fig. 5 (e) and (f), leads to the conclusion that the viscoelastic dampers can be more effective than viscous dampers for the displacement control of asymmetric structures.

5. Optimum distribution of supplemental dampers

The results from the parametric study indicate that there exists a damper eccentricity which minimizes the torsional response of a single-storey asymmetric structure. In this study a simple procedure based on the modal characteristics of asymmetric structure is proposed to find out the appropriate eccentricity of the supplemental dampers.

5.1. Modal characteristics of a structure with stiffness eccentricity

Table 1 describes the change in both the lateral and the rotational components of the first and the second mode shape vectors of the model structure without added dampers for various stiffness eccentricities $\bar{e}$. It can be observed that the lateral and rotational components are independent of each other when there is no eccentricity, whereas they appear simultaneously when the stiffness eccentricity exists. The degree of connection becomes stronger as the eccentricity increases, which indicates that the modal components are closely related to the torsional effect caused by the plan-wise asymmetry. To quantify the relation between the lateral and the torsional motion, the ratio of the real parts of the rotational and the lateral components of the fundamental mode is denoted as R:

$$R = \frac{\Phi_\theta}{\Phi_y} \tag{4}$$

Table 1 also shows that the lateral or rotational behavior become dominant as R is getting close to 0 or to $\propto$, respectively. Also it can be noticed that R plays an important role representing the torsional property of the structure; i.e. the left-hand-side is flexible when R is negative, and the opposite is true when R is positive as shown in Fig. 8.

Based on the observation it can be expected that with proper plan-wise distribution of the supplemental dampers it would be possible to find the point that the modal coefficient ratio R becomes nearly zero. In this study this point will be called the point of *Optimum Damper Eccentricity* (ODE).

Fig. 9 plots the maximum displacements of the flexible and stiff edges of the model structure installed with both viscous and viscoelastic dampers, obtained from time history analysis with El Centro earthquake, NS component. The stiffness eccentricity was set to be 0.2 and both dampers were designed to have the same damping coefficients. The inherent damping of the structure was assumed to be 5%, and the fundamental modal damping ratio of the equivalent symmetric structure turned out to be 33% after the dampers were installed. Fig. 9 shows that as the eccentricity $\bar{e}_{sd}$ of the supplemental viscoelastic damper increases, the maximum displacement of the stiff edge increases and that of the

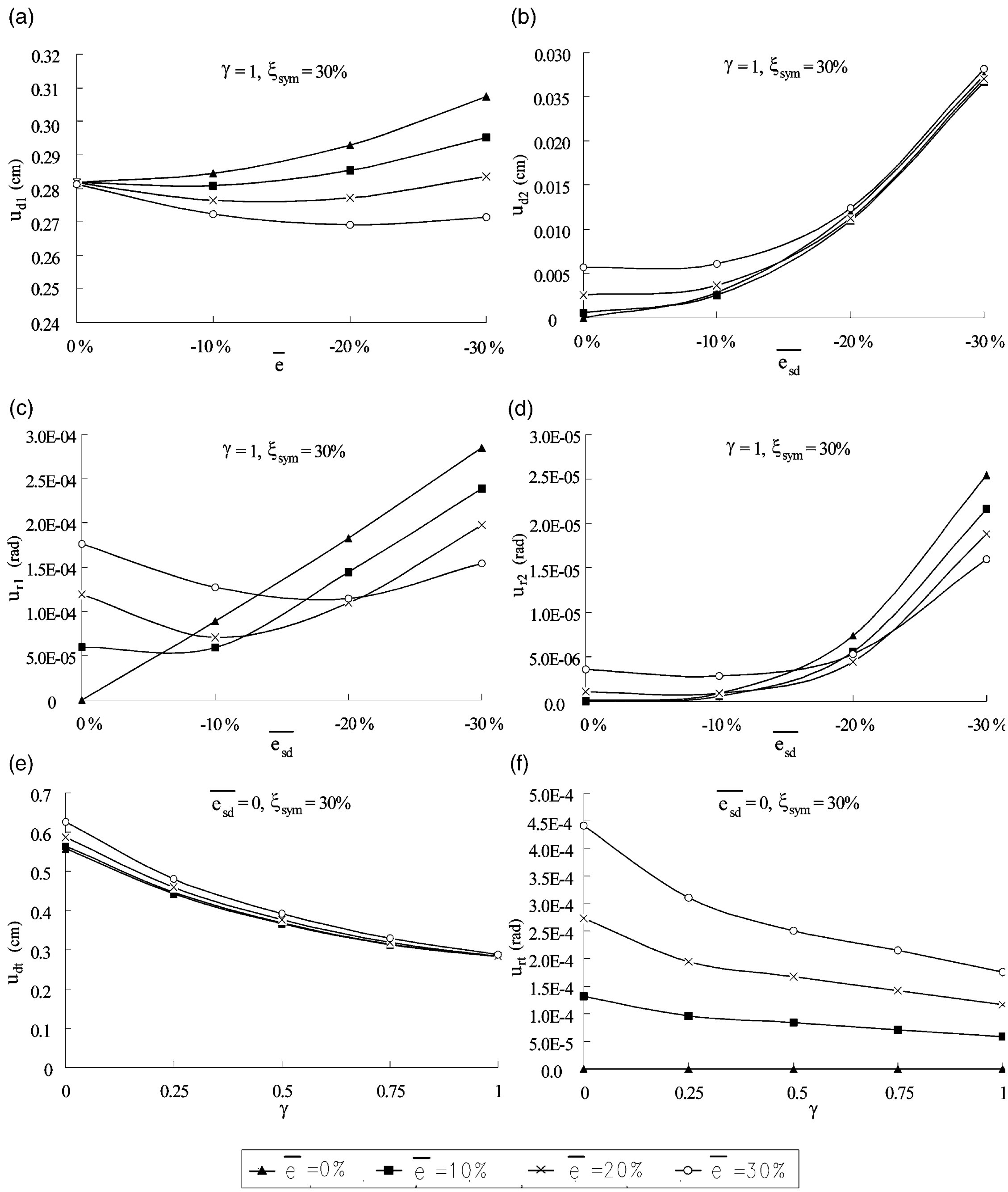

Fig. 5. Modal displacements and rotation at CM(u_d,u_r).

flexible edge decreases, and that finally the displacements of the two edges meet at one point. The point is considered to be the ODE. However when it comes to viscous dampers it can be observed that the maximum displacements of both edges change only slightly and fail to become identical for the given damping eccentricities.

According to the results of the modal analysis presented in Fig. 10, the value of R reaches near zero at the point that the maximum displacements of the two edges meet. Also if the damper eccentricity exceeds the ODE, the rotation of the plan occurs in the opposite direction. Consequently even before the response analysis has been carried out, the point of optimum damping distribution may approximately be determined from modal analysis. The ODE determined from modal analysis can be considered as a more generalized solution since it is a characteristic of the given structure, while the result obtained from response analysis varies depending on the earthquake records used.

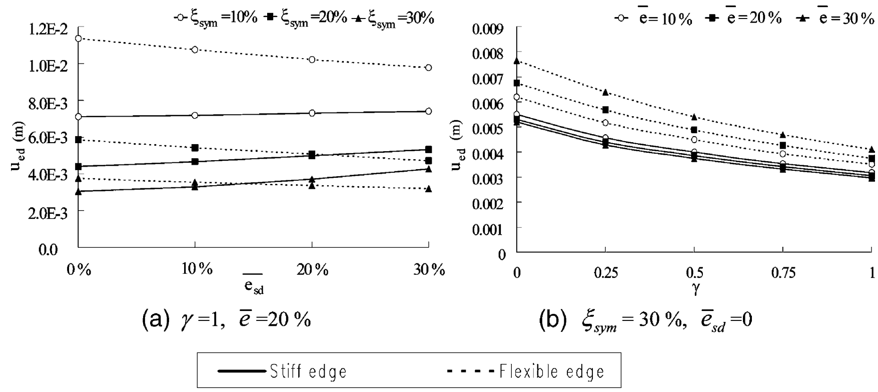

Fig. 6. Edge displacements for El Centro earthquake: (a) $\gamma = 1$, $\bar{e}_{sd} = 0$; (b) $\gamma = 1$, $\bar{e} = 0.2$; and (c) $\xi_{sym} = 0.3$, $\bar{e}_{sd} = 0$.

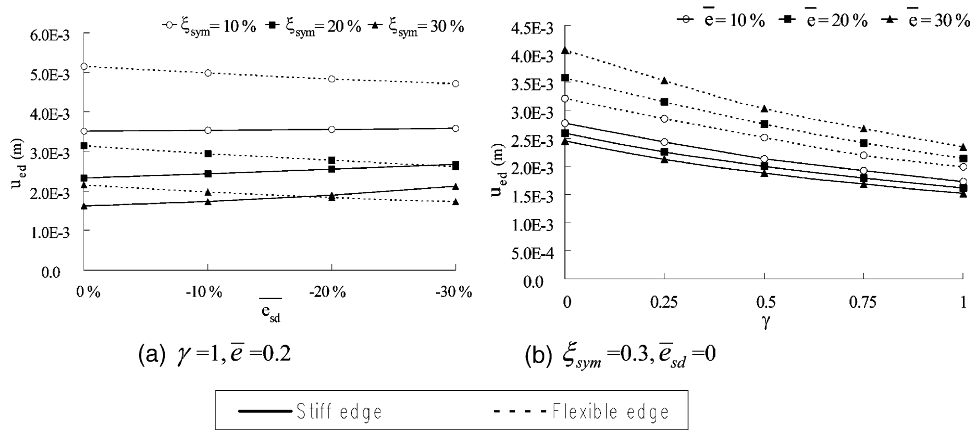

Fig. 7. Edge displacements for Taft earthquake: (a) $\gamma = 1$, $\bar{e}_{sd} = 0$; (b) $\gamma = 1$, $\bar{e} = 0.2$; and (c) $\xi_{sym} = 0.3$, $\bar{e}_{sd} = 0$.

Table 1
Change in the modal coefficients and their ratio R for stiffness eccentricity

$\bar{e}$		0	0.1	0.2	0.3
1st mode	Φ_y	-0.3588	0.3513	0.3415	0.3333
	Φ_θ	0	−0.1417	−0.2547	−0.3315
	R	0	−0.4035	−0.7458	−0.9945
2nd mode	Φ_y	0	−0.0198	-0.0365	−0.0486
	Φ_θ	−0.2628	−0.5602	−0.2527	−0.2443
	R	∝	13.14	6.9233	2.835

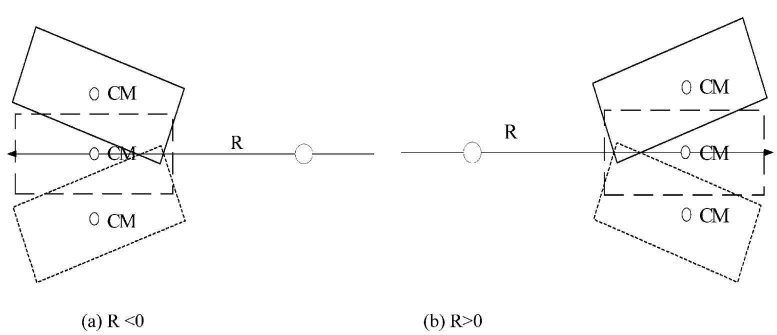

Fig. 8. Behaviour of a plan-asymmetric structure with change in R factor.

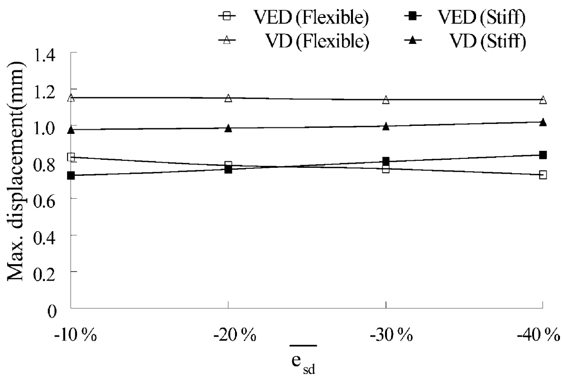

Fig. 9. Maximum displacements of the stiff and the flexible edges.

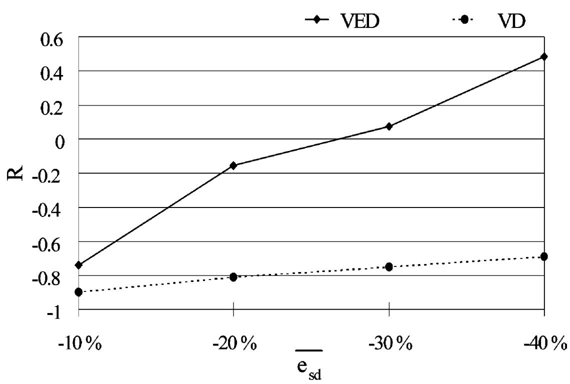

Fig. 10. Change in R for damper eccentricity.

5.2. Diagram for optimum damper eccentricity

Fig. 11 presents the change in the ODE as a function of ω_y for various values of Ω, the rotational to translational natural frequency ratio. Both the stiffness eccentricity and the modal damping ratio of the corresponding symmetric system were set to be 20%, and γ=0.95 was assumed for the viscoelastic material. It can be observed in the figure that the ODE is not affected by ω_y as long as Ω is constant, and that the ODE tends to increase as Ω increases.

The results presented in Figs. 9–11 indicate that parameters such as $\bar{e}$, $\bar{e}_{sd}$, and Ω play an important role in the determination of ODE. Based on this observation the points of optimum damper eccentricity for the 2-DOF structure were plotted in Fig. 12 as a function of stiffness eccentricity $\bar{e}$ and natural frequency ratio Ω for a constant modal damping ratio ξ_{sym}=30%. The points on the ODE curve corresponds to where the R factor becomes zero. It can be noticed in the diagram that the ODE increases as $\bar{e}$ and Ω increase. The damping ratio ξ_{sym} may be determined from the consideration of mitigating overall seismic responses. If such diagrams are constructed in advance for practical range of damping ratios, the ODE associated with a structure with asymmetric stiffness distribution can easily be determined from the diagram, and the torsional as well as translational dynamic responses can be significantly reduced by appropriate placement of supplemental dampers.

5.3. Determination of damper size in multi-storey structures

The present study requires information for an appropriate storey-wise distribution of supplemental viscoelastic dampers to realize the given modal damping ratio. For a single-storey structure it is not complicated to determine the size of the dampers corresponding to a certain modal damping ratio. For a multi-storey structure, however, it is not so straightforward to achieve a specific (fundamental) modal damping ratio, and basically an iterative process is required. In this study the required amount of the supplemental dampers were obtained without iteration by utilizing the concept of equivalent damping, which can be obtained from the ratio of the dissipated energy E_D and the stored strain energy of the structure E_S as follows [8]:

$$\xi_{eq} = \frac{E_D}{4\pi E_S} \tag{5}$$

For simplicity, it is assumed that the structure vibrates only in accordance with the fundamental vibration mode, and that the system behaves within linear elastic range.

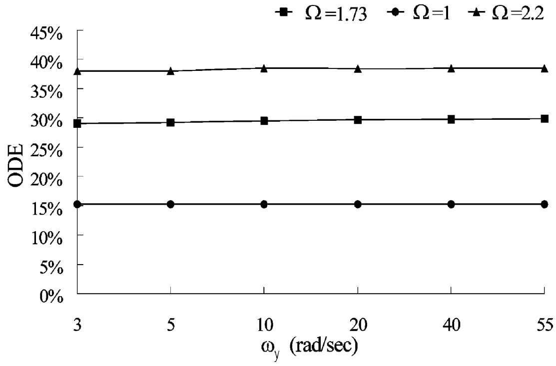

Fig. 11. Change in ODE as a function of frequency ω_y and frequency ratio Ω.

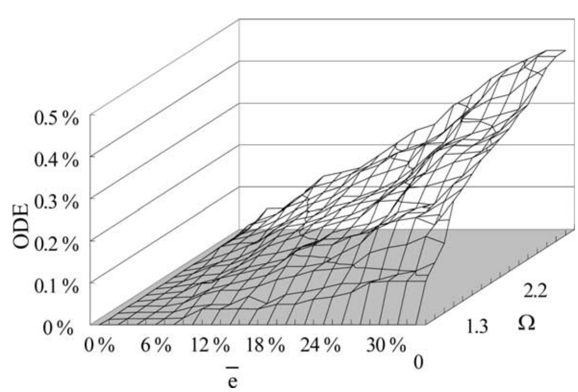

Fig. 12. ODE diagram constructed for damping ratio ξ_{sym}=0.3.

Then the dissipated energy is mostly contributed from the added dampers, which can be computed as follows [12]:

$$E_D = \frac{2\pi^2}{T}\sum_j c_{dj}\cos^2\theta_j(\Delta_j - \Delta_{j-1})^2 \quad (6)$$

where θ_j, c_j, and Δ_j are the inclined angle of the dampers, the damping coefficient of the dampers, and the story drift at the jth story, respectively, and T is the fundamental natural period of the structure with the dampers. Similarly the strain energy of the structure with the VED can be expressed as follows:

$$E_S = \frac{2\pi^2}{T^2}\sum_j m_j\Delta_j^2 + \frac{1}{2}\sum_j k_{dj}\cos^2\theta_j(\Delta_j - \Delta_{j-1})^2 \quad (7)$$

where m_j is the mass of the jth story, and k_{dj} is the stiffness of the dampers located in the jth story. The first term in the right-hand-side of Eq. (7) represents the strain energy stored in the structure, and the second term is the strain energy stored in VED. By substituting the above two equations into Eq. (5), and using the components of the fundamental mode shape vector instead of the story drifts (which are unknown in this stage), the equivalent viscous damping ratio can be expressed as follows:

$$\xi_{eq} = \frac{1}{4\pi}\frac{\frac{2\pi^2}{T}c_d\sum_j \cos^2\theta_j(\Phi_j - \Phi_{j-1})^2}{\frac{2\pi^2}{T^2}\sum_j m_j\Phi_j^2 + \frac{1}{2}k_d\sum_j \cos^2\theta_j(\Phi_j - \Phi_{j-1})^2} \quad (8)$$

where it was assumed that the damper properties are the same in every story. Also the following relationship between the stiffness and damping coefficients of the VED can be derived from Eq. (1):

$$k_d = \frac{G'\omega}{G''}c_d \quad (9)$$

By substituting Eq. (9) into Eq. (8), the unknown damping coefficient of the viscoelastic dampers can be obtained for the given equivalent damping ratio, and the damper size can be determined from the damping coefficient and the properties of the viscoelastic material used. The next step is to distribute the dampers both in stiff and flexible edges in such a way that the R factor computed from the components of the fundamental mode shape vector becomes close to zero.

5.4. Design procedures

With the information acquired up to this point, we can determine the required amount of viscoelastic (or viscous) dampers both for flexible and stiff edges based on the following procedure:

1. From the corresponding symmetric structure, determine the damping ratio ξ_{sym} required to be supplemented to satisfy the desired performance objective.
2. Determine the amount of dampers that can realize the required damping ratio from Eq. (8).
3. Distribute the dampers on the stiff and the flexible edges eccentrically, and carry out an eigenvalue analysis. From the components of the fundamental mode shape vector, obtain R factor (or factors in a multi-story structure) from Eq. (4).
4. Repeat the procedure (3) with different damper eccentricity until the R factor becomes nearly zero. This eccentricity is closely related to the optimal damper eccentricity that minimize the torsional response of the plan-wise asymmetric structure.

Alternatively, if the ODE diagram is already constructed,

1. Determine the fundamental translational and the rotational natural frequencies and their ratio, Ω, from the eigenvalue analysis of the original asymmetric system without dampers.
2. Read the optimal damper eccentricity from the ODE diagram with the given Ω and the stiffness eccentricity.

6. Numerical example

6.1. Single-storey model structure

To verify the usefulness of the ODE diagram, the responses of the single-story model structure with the following parameters were obtained for the El Centro and Taft earthquake ground motions: — case 1: Ω=1.5, $\bar{e}$=0.2, — case 2: Ω=2.27, $\bar{e}$=0.3. The viscoelastic dampers with the ratio of the storage and the loss modulus, γ, equal to 0.95 were installed so that the damping ratio of the structure became 30% of the critical damping including the 5% of inherent viscous damping. It can be observed in the ODE diagram (Fig. 12) that the ODE corresponds to about 0.16 and 0.3 for the case 1 and 2, respectively. This implies that by installing the dampers with such eccentricities the torsional responses of the structures are nearly minimized. Fig. 13 presents the displacements of the structure with various damper eccentricities. From the figures it can be noticed that the displacements of the flexible and stiff edges become almost identical when the damping eccentricities are 0.18 and 0.3 for El Centro earthquake, and 0.17 and 0.27 for the Taft earthquake. These correspond well with the ODE of 0.16 and 0.3 determined from the ODE diagram. This seems to be quite satisfactory considering the fact that the ODE diagram was constructed based only on the modal characteristics of the structure.

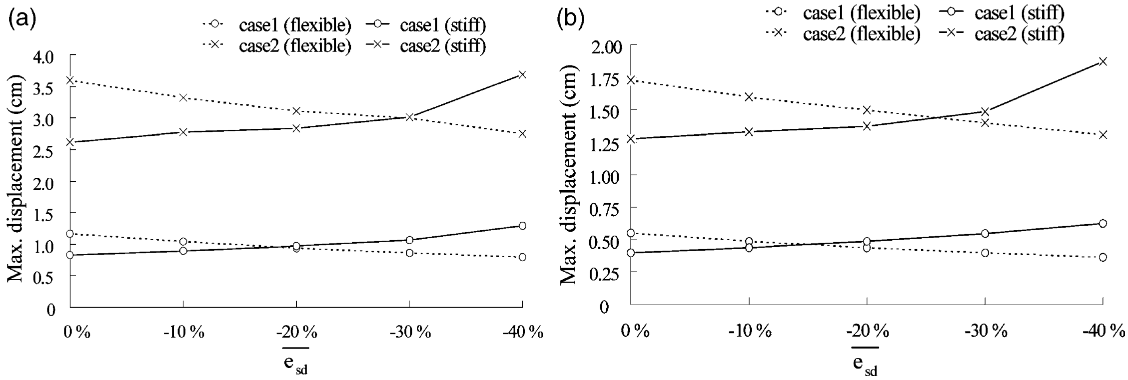

Fig. 13. Maximum displacement responses for various damping eccentricities.

6.2. 5-storey structure

The proposed procedure was applied to a five-storey shear building shown in Fig. 14. Only two degrees of freedom, translation in the Y-dir. and rotation, were considered for each floor. The columns were selected in such a way that the stiffness eccentricity $\bar{e}$ became 0.3. A pair of viscoelastic dampers with γ=0.95 were placed on every floor, and the size of the dampers was determined so that the fundamental modal damping ratio ξ_{sym} reached 30%. The ratio of the fundamental rotational and translational natural frequencies Ω turned out to be 0.24. Based on these information the ODE of 0.3 was obtained from the ODE diagram shown in Fig. 12 and the dampers were asymmetrically distributed on the plan.

Table 2 shows that the modal participation of the fundamental mode is dominant even with the 30% stiffness eccentricity. Therefore in this study the modal coefficient ratio *R* was obtained from the coefficients of the first mode shape vector. Table 3 shows the variation of *R* in each storey for various damping eccentricities. It can be observed that in every story the sign of *R* factor changes between the damper eccentricity of 20% and 30%. Fig. 15 indicates that *R* factors become zero between the damper eccentricity of 27–30%. Fig. 16 (a) and (b) represent the maximum displacements of the stiff and the flexible edges for El Centro and Taft earthquake (NS components), respectively. It can be observed that the displacements at both edges become identical when the damper eccentricity is between 25–30%. Therefore the ODE determined from the ODE diagram and from Fig. 16 using the coefficients of the fundamental mode seems to be valid.

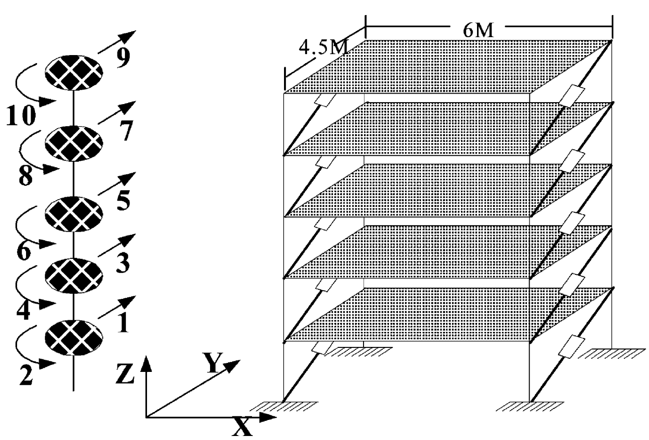

Fig. 14. Multi-storey model structure.

Table 2
Modal participation factors

Mode	Modal Participation Factors(Γ_n^*)	Participation Rates (%)
1	10.5802	86.5
2	1.0356	8.46
3	0.4808	3.94
4	0.0394	0.32
5	0.0662	0.54
6	0.0188	0.16
7	0.0024	0.02
8	0.0068	0.06
9	0.0002	0.00
10	0.0024	0.02

7. Conclusion

In this study a strategy for an appropriate plan-wise distribution of viscoelastic dampers was proposed based on modal characteristics of a structure-damper system to minimize the torsional responses of a structure with only one axis of symmetry. It was found that appropriate plan-wise distribution of supplemental viscoelastic dampers can effectively reduce the torsional response of an asymmetric structure. However, it should be pointed out that the proposed method can only be applicable for structures deforming within linear elastic range, since it is based on the modal characteristics of the structure. More specific results derived from the parametric study and the response analysis of model structures are summarized as follows:

Table 3
Change in the R factor for damping eccentricities ($\bar{e}$=0.3)

Story	DOF	$\bar{e}_{sd} = 0$ 1st mode shape vector	R_n	$\bar{e}_{sd} = -0.1$ 1st mode shape vector	R_n	$\bar{e}_{sd} = -0.2$ 1st mode shape vector	R_n	$\bar{e}_{sd} = -0.3$ 1st mode shape vector	R_n	$\bar{e}_{sd} = -0.4$ 1st mode shape vector	R_n
1	1	−0.31	0.64	−0.31	0.33	−0.32	0.10	−0.32	0.0	−0.27	−0.04
	2	−0.20		−0.10		−0.03		0.00		0.01	
2	3	−0.58	0.68	−0.61	0.34	−0.6	0.12	−0.6	0.0	−0.54	−0.062
	4	−0.39		−0.21		−0.65		0.00		0.03	
3	5	−0.78	0.72	−0.82	0.37	−0.82	0.13	−0.81	−0.01	−0.79	−0.83
	6	−0.57		−0.30		−0.11		0.01		0.07	
4	7	−0.91	0.75	−0.96	0.41	−0.96	0.14	−0.95	−0.03	−0.99	−0.11
	8	−0.69		−0.39		−0.13		0.03		0.11	
5	9	−1	0.81	−1	0.43	−1	0.15	−1	−0.04	−1	−0.13
	10	−0.81		−0.42		−0.15		−0.04		0.13	

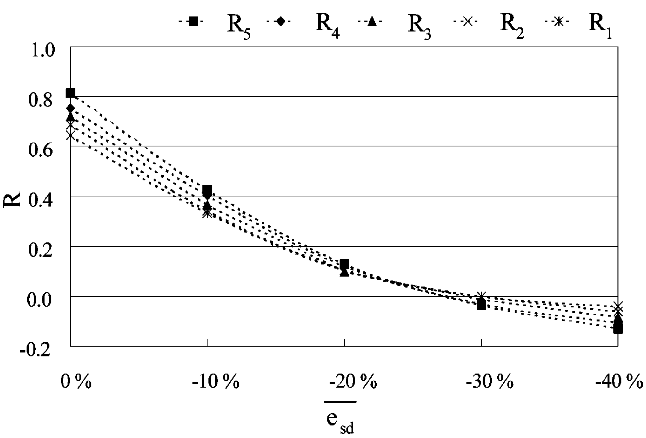

Fig. 15. Change of R-factor in each story for various damper eccentricities.

1. The natural frequencies of a plan-wise asymmetric structure are not affected by the change in the damper eccentricity $\bar{e}_{sd}$ and the stiffness eccentricity $\bar{e}$, but are affected by the ratio of storage to loss modulus of the viscoelastic materials γ and the damping ratio.
2. The modal damping ratios are not affected by the stiffness eccentricity $\bar{e}$, but are more or less affected by the change in $\bar{e}_{sd}$.
3. The overall response decreases as the stiffness of the damper increases. Also the displacements at stiff and flexible edges become equal at smaller damping eccentricity when the stiffness of the dampers is larger. Therefore for the purpose of reducing the displacement response of a structure caused by the plan-wise eccentricity, the performance of the viscoelastic dampers are considered to be superior to that of the viscous dampers.
4. For multi-storey structures the optimum damping eccentricity may be estimated based on the coefficients of the fundamental mode shape vector obtained from eigenvalue analysis of nonproportionally-damped structure with added dampers.

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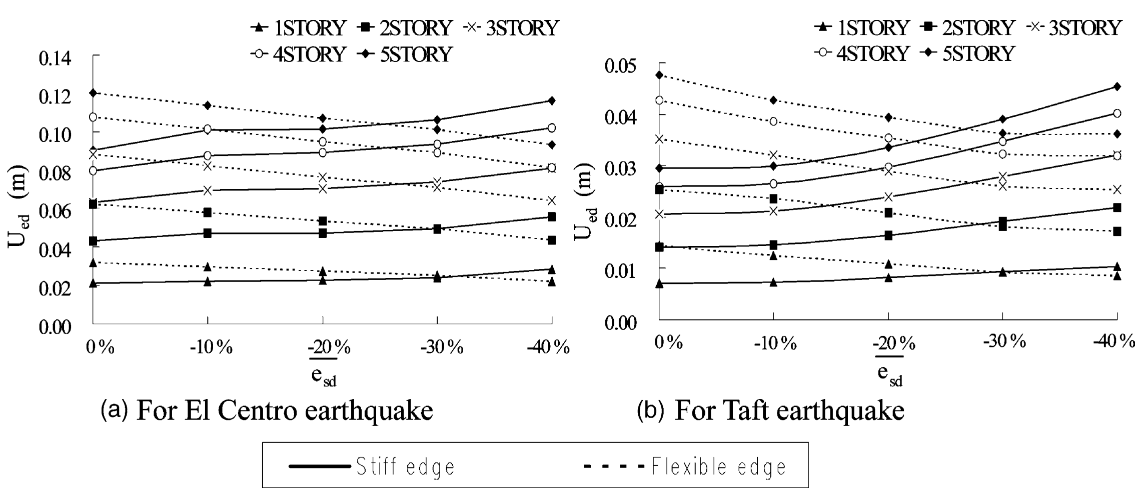

Fig. 16. Edge displacements of the 5-storey structure.

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Appendix

In this appendix the components of the stiffness matrix of the combined structure-viscoelastic dampers were derived with the parameters of the structure and the viscoelastic dampers. To this end the shear storage modulus G' and the loss modulus $G''$are expressed as follows utilizing Eq. (1) and the dynamic characteristics of the single-storey structure:

$$G' = \frac{2mt\xi\omega_y^2}{A}\gamma \, G'' = \frac{2mt\xi\omega_y^2}{A} \quad \text{(A1a, b)}$$

where γ is the ratio of the storage and the loss moduli, and the natural frequency of the structure ω_y is used instead of the forcing frequency $\bar{\omega}$ on the assumption that the dynamic responses are dominated by the fundamental mode. Substituting Eq. A1a into Eq. (1) leads to the following expression for the stiffness of the dampers:

$$k_d = 2\, m\, \xi\, \omega_y^2\, r = 2\, \xi\, k_t\, r \quad \text{(A2)}$$

where $k_t = k_o + k_d$; k_t is the total lateral stiffness of the structure including the dampers, k_o is the lateral stiffness of the structure at the mass center; k_d is the stiffness of the supplemental viscoelastic dampers, ξ is the damping ratio of the corresponding symmetric structure system. In this study the inherent damping of the structure was neglected in the formulation since the inherent damping of an elastic structure is much smaller than that supplied by the added dampers. The system damping was expressed as ξ_{sym} in the preceeding sections. Finally, ω_y is the natural frequency of the structure after the dampers are installed. The above equation leads to the following relation between the stiffness of the dampers and the structure:

$$k_d = \frac{2\, \xi\, \gamma}{1-2\, \xi\, \gamma} k_o \quad \text{(A3)}$$

By substituting Eq. A3 back into Eq. A2 the relation between the stiffness of the structure before and after the dampers are installed can be expressed as follows:

$$k_t = \mu\, k_o \qquad \mu = \frac{1}{1-2\, \xi\, \gamma} \quad \text{(A4a, b)}$$

where μ is named as the *stiffness factor*. If γ is equal to zero, which is the case of viscous dampers, the stiffness factor μ becomes a unit value.

To reduce the number of variables, the ratio of rotational and translational natural frequencies Ω is defined as follows for the given model structure:

$$\Omega = \frac{\omega_\theta}{\omega_y} = \sqrt{\frac{mk_\theta}{m_\theta k_y}} = \sqrt{\frac{3\lambda^2 + 3\alpha^2}{\alpha^2 + 1}} \quad \text{(A5)}$$

where k_y and k_θ are the stiffness along y and θ directions, respectively, and $m_\theta = \frac{m(a^2 + d^2)}{12}$, $k_\theta = k_x(a/2)^2 + k_y(d/2)^2$, and $\lambda = \omega_x/\omega_y$. Using the above relation the aspect ratio of the floor, $\alpha = a/d$, can be denoted by Ω and the natural frequency ratio of x and y directions:

$$\alpha^2 = \frac{3\lambda^2-\Omega^2}{\Omega^2-3} \quad \text{(A6)}$$

The above equation leads to the following constraints: $\sqrt{3} < \Omega < \sqrt{3}\lambda$ when $\lambda > 1$, $\sqrt{3}\lambda < \Omega < \sqrt{3}$ when $\lambda < 1$, and $\Omega = \sqrt{3}$ when $\lambda = 1$. It can be noticed that Ω can have only finite values depending on λ.

Next, the stiffness eccentricity and frequency ratio of the system with viscoelastic dampers are derived. Generally the stiffness eccentricity can be written as follows:

$$\bar{e} = \frac{k_{sy}-k_{fy}}{2(k_{sy} + k_{fy})} = \frac{k_{sy}-k_{fy}}{2\omega_y^2 m} \quad \text{(A7)}$$

where k_{sy} and k_{fy} are the y-direction stiffness of the stiff edge and the flexible edge, respectively. The term $k_{sy}-k_{fy}$ can be obtained by adding the stiffness of the structure and the dampers:

$$k_{sy}-k_{fy} = 2e_s\omega_{yo}^2 m + 2e_d\omega_d^2 m \quad \text{(A8)}$$

where ω_{yo}, and e_s are the natural frequency in the y-direction and the stiffness eccentricity of the structure before the dampers are installed, respectively, and ω_d and e_d are those of the dampers, respectively. Therefore the eccentricity after the dampers are installed becomes:

$$\bar{e_t} = \frac{e_s\omega_{yo}^2 + e_d\omega_d^2}{\omega_y^2} \quad \text{(A9)}$$

Eq. (A5) and (A6) leads to:

$$\omega_y = \omega_{yo}\sqrt{\frac{1}{1-2\xi\gamma}} \qquad \omega_d = \omega_{yo}\sqrt{\frac{2\xi\gamma}{1-2\xi\gamma}} \quad \text{(A10)}$$

By substituting Eq. (A10) into (A9) the following equation can be obtained:

$$\bar{e_t} = \left[\bar{e} + \bar{e}_{sd}\left(\frac{2\xi\gamma}{1-2\xi\gamma}\right)\right](1-2\xi\gamma) \quad \text{(A11)}$$

On the other hand the torsional-translational frequency ratio after the dampers are installed, Ω_t, can be obtained from the following relation:

$$\frac{3\lambda^2-\Omega^2}{\Omega^2-3} = \frac{3\lambda_t^2-\Omega_t^2}{\Omega_t^2-3} \quad \text{(A12)}$$

where λ_t is the λ modified by the dampers. By substituting the relation $\lambda_t^2 = \lambda^2\frac{1}{\mu}$ into the above equation Ω_t is expressed as follows:

$$\Omega_t = \frac{(\lambda^2-2\lambda^2\xi\gamma-1)\Omega^2 + 6\lambda^2\xi\gamma}{\lambda^2-1} \quad \text{(A13)}$$

The stiffness matrix of the model structure with added viscoelastic dampers can be constructed using Eq. (2), (A10), (A11), and (A13):

$$K = \frac{m\omega_y^2}{\mu}\begin{bmatrix} k_{11} & k_{12} \\ k_{21} & k_{22} \end{bmatrix} \tag{A14}$$

where $k_{11} = \frac{m\omega_y^2}{\mu}$, $k_{12} = m\omega_y^2(\bar{e} + \bar{e}_{sd}\frac{2\xi\gamma}{\mu})$, $k_{21} = m\omega_y^2(\bar{e} + \bar{e}_{sd}\frac{2\xi\gamma}{\mu})$, and $k_{22} = \frac{m\omega_y^2}{\mu} \times \left[((\bar{e} + \bar{e}_{sd}\frac{2\xi\gamma}{\mu})\mu)^2 + \frac{(\lambda^2-2\lambda^2\xi\gamma-1)\Omega^2 + 6\lambda^2\xi\gamma}{12\gamma^2-4\Omega^2}\right]$

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