Equivalent damping of a structure with vibration control devices subjected to wind loads

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Abstract. The purpose of this study is to propose a procedure for evaluating quantitatively the increase of the equivalent damping ratio of a structure with passive/active vibration control systems subjected to a stationary wind load. A Lyapunov function governing the response of a structure and its differential equation are formulated first. Then the state-space equation of the structure coupled with the secondary damping system is solved. The results are substituted into the differential equation of the Lyapunov function and its derivative. The equivalent damping ratios are obtained from the Lyapunov function of the combined system and its derivative, and are used to assess the control effect of various damping devices quantitatively. The accuracy of the proposed procedure is confirmed by applying it to a structure with nonlinear as well as linear passive/active control systems.

Keywords: equivalent damping; wind load; structural control; active/passive dampers.

1. Introduction

The quantification of damping is complicated when a passive/active mechanical damping device is installed in the structure. For the purpose of investigating the control effects of added damping devices, many researchers adopted the concept of equivalent damping ratio (EDR). Hartog (1956)

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calculated the increase of modal damping ratios of a primary structure with a tuned mass damper (TMD) as a function of mass ratios. Johnson and Kienholz (1982), Soong and Lai (1991), and Chang *et al*. (1992) applied the modal-strain-energy method to assess the effect of a viscoelastic damper (VED). Li and Reinhorn (1995) derived the damping ratios of a structure with supplemental friction dampers through an identification procedure using acceleration response transfer functions.

The damping ratio contributed from a linear supplemental damper can be estimated precisely by eigenvalue analysis. However in a structure with nonlinear damping devices, which impose nonlinear control force on the structure, the eigenvalue analysis cannot be applied. The nonlinear damping devices include passive devices, such as friction dampers and hysteretic devices, and active control devices subjected to control force saturation or stroke saturation. Even in devices categorized as linear systems, nonlinear control forces can be generated as a result of temperature change, heat generated by cyclic behavior, or friction between the device and the structure. Therefore a convenient but accurate method to estimate the damping ratio supplied by nonlinear supplemental dampers needs to be developed.

The purpose of this study is to propose an approach to evaluate the EDR of a structure with any supplemental vibration control devices subjected to a stationary wind load. The proposed method has an advantage in that the equivalent damping of a structure with nonlinear added dampers can be estimated, for which an eigenvalue analysis cannot be applied. To show the effectiveness of the proposed approach, the EDRs of a structure with viscous dampers (VD), a tuned mass damper (TMD), an active mass driver (AMD), and friction dampers (FD) subjected to a wind load are computed. For validation of the proposed method, the results from the proposed method for linear system are compared with those obtained from eigenvalue analysis. In this study, it is assumed that the response of a structure is a stationary random process and the control device does not affect the mode shape of the structure.

2. Derivation of equivalent damping ratios

2.1. Lyapunov function for general energy

In this section and the following section, the Lyapunov function whose derivative is expressed in autoregressive form is obtained in modal space as a general energy form. The EDR is evaluated through the time history analysis using the Lyapunov function and its derivative.

The equation of motion of a structure without a damping device can be written as follows:

$$M\dot{x} + C\dot{x} + Kx = f(t) \tag{1}$$

where M, C, and K are the mass, damping, and stiffness matrix, respectively; x is a displacement vector; and $f(t)$ is a disturbance. Eq. (1) can be transformed to the following state-space equation:

$$\dot{z} = Az + B_1 f(t) \tag{2}$$

where $z=[x \;\; \dot{x}]^T$, and the system matrix, A, and the location matrix of disturbance, B_1, can be represented as:

$$A = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}C \end{bmatrix} \qquad B_1 = \begin{bmatrix} 0 \\ M^{-1} \end{bmatrix} \tag{3}$$

In this study the energy of a structure is defined as follows:

$$e = z^T Q_o z \tag{4}$$

where Q_o, the energy matrix, has the following form:

$$Q_o = \begin{bmatrix} K & 0 \\ 0 & M \end{bmatrix} \tag{5}$$

Strictly speaking, the energy matrix generally used is a half of Eq. (5), but it is defined as Eq. (5) for simplicity in mathematical formulation. As the energy in a structure corresponds to a special form of a Lyapunov function, Eq. (4) can be written as the generalized form of a Lyapunov function:

$$e = z^T Q z \tag{6}$$

where Q is a positive definite matrix satisfying conditions for the Lyapunov function. In this case e has the dimension of energy, and can be called a 'generalized energy'. By differentiating both sides of Eq. (6) and substituting Eq. (2), we get

$$\dot{e} = z^T (A^T Q + QA) z + f^T B_1^T Q z + z^T Q B_1 f \tag{7}$$

If the first term in the right-hand-side of Eq. (7) satisfies the following equation:

$$A^T Q + QA = -\alpha Q \tag{8}$$

then Eq. (7) can be reduced to

$$\dot{e} = -\alpha e + F(t) \tag{9}$$

where α is a positive scalar, and $F(t)$ is the energy dissipated by the disturbance f, which is expressed as follows:

$$F(t) = f^T B_1^T Q z + z^T Q B_1 f \tag{10}$$

If a matrix Q and a scalar α which satisfy the Lyapunov equation do exist, Eq. (9) becomes the 1st order differential equation with regard to the generalized energy, e. The scalar α determines the rate of convergence of the generalized energy. The fact that α is always positive guarantees the system stability, since the energy is always decreased. Therefore it can be conjectured that α is a parameter related with structural damping.

Generally in a MDOF system, there is no matrix Q that satisfies Eq. (8); the matrix Q becomes the right eigen matrix and the left eigen matrix at the same time, and there can exist no two identical eigen matrices related to an asymmetric system matrix, A. However if it is assumed that there exists a dominant mode and the other modes have little effect on the response of the structure, we can obtain the matrix Q and scalar α in modal space satisfying Eq. (8) as follows:

$$Q_i = \begin{bmatrix} w_i^2 & \xi_i w_i \\ \xi_i w_i & 1 \end{bmatrix} \qquad \alpha_i = 2\xi_i w_i \tag{11}$$

where w_i and ξ_i are the natural frequency and damping ratio of the ith mode, respectively. In the above equation it can be observed that λ_i is the product of the damping ratio and the natural frequency of the ith mode. The modal energy and the 1st order differential equation of the ith modal energy can be written as:

$$e_i = z_i^T Q_i z_i \tag{12}$$

$$\dot{e}_i = -\alpha_i e_i + F_i(t) \tag{13}$$

where $F_i(t)$ is the rate of change in energy due to the external load:

$$F_i(t) = f_i^T B_{1i}^T Q_i z_i + z_i^T Q_i B_{1i} f_i \tag{14}$$

where f_i is the generalized external load acting on the ith mode, and B_{1i} and z_i are represented as

$$B_{1i} = [0 \quad 1]^T, \qquad z_i = [\eta_i \quad \dot{\eta}_i]^T \tag{15}$$

where η_i and $\dot{\eta}_i$ are the generalized displacement and velocity of the ith mode, respectively.

2.2. Equivalent damping ratio in modal space

The equation of motion of a structure with an added damping device can be written as

$$M\dot{x} + C\dot{x} + Kx = f(t) + Lu(t) \tag{16}$$

where the matrix L represents the location of the control force, which depends on the type of damping device, and $u(t)$ is the generalized control force. The differential equation for energy generated from the control force in modal space is expressed as

$$\dot{e}_i = -\alpha_i e_i + U_i(t) + F_i(t) \tag{17}$$

where $U_i(t)$ is defined as the equivalent control energy, which is the energy dissipated by the control force:

$$U_i(t) = u_i^T B_{2i}^T Q_i z_i + z_i^T Q_i B_{2i} u_i \tag{18}$$

where u_i is the generalized control force applied to the ith mode and $B_{2i} = [0 \quad 1]^T$ determines the location of the control force.

Considering the fact that the parameter α_i in Eq. (17) represents the decrease of energy due to the inherent damping, the equivalent control energy, $U_i(t)$, can be written as follows using another parameter, β_i:

$$U_i(t) = -\beta_i e_i(t) \tag{19}$$

where β_i is the function of time because Lyapunov function and equivalent control energy are also functions of time. If $Ui(t)$ and e_i are stationary random processes such as wind-induced vibration, the parameter β_i can be obtained by taking expectation of both sides of Eq. (19):

$$\beta_i = -\frac{E[U_i(t)]}{E[e(t)]} \tag{20}$$

where $E[.]$ is the function of expectation. The increase in the equivalent damping ratio by the addition of the damping device can be defined as follows using the parameter β_i:

$$\xi_{i_{eq}} = \frac{\beta_i}{2w_i} \tag{21}$$

Finally the modal equation of motion can be converted to the following equation using the equivalent damping ratio $\xi_{i_{eq}}$:

$$\ddot{\eta}_i + 2(\xi_i + \xi_{i_{eq}})w_i\dot{\eta}_i + w_i^2\eta_i = f_i(t) \tag{22}$$

3. Modal equivalent damping ratios for various damping devices

In this section, analytical approach to evaluate the modal EDR has been developed for various types of damping devices, such as linear viscous dampers (LVD), a tuned mass damper (TMD), an active mass damper (AMD), and Coulomb friction dampers (FD-friction damper). The first three dampers are linear damping devices, and the last one is the nonlinear damping device. In case of FD, a closed form formula for equivalent damping is derived. The model structure for analysis is a 10-story shear building with damping devices subjected to a wind load. The dynamic properties of the model structure are presented in Table 1. It is assumed that the damping matrix of the shear building is proportional to the mass and the stiffness matrices (i.e., proportional damping), and the wind disturbance is a stationary random process.

3.1. Modeling of wind load

As the proposed procedure estimates equivalent damping ratios using structural responses, a mathematical formulation of a external wind load is required. The wind load at each story is generally correlated each other; however for ease of computation it is assumed that the wind load in each story is independent at each story. This simplification may be justified considering that the primary goal of the paper is to present a procedure for estimating equivalent damping ratio contributed from added dampers excited by any type of dynamic load. In this study it is assumed that the vertical distribution of wind load acting on the model structure follows the power law as

Table 1 Modal characteristics of the model structure

Properties	Values		
Story mass	m = 78.75 ton		
Story damping	c = 177 kN sec/m		
Story stiffness	k = 34,800 kN/m		
Mode shape vectors	1st	2nd	3rd
	0.1495	-0.4450	0.6821
	0.2956	-0.8019	1.0000
	0.4351	-1.0000	0.7840
	0.5649	-1.0000	0.1495
	0.6821	-0.8019	-0.5649
	0.7840	-0.4450	-0.9777
	0.8685	0.0000	-0.8685
	0.9335	0.4450	-0.2956
	0.9777	0.8019	0.4351
	1.0000	1.0000	0.9335
1st modal mass	415.76 ton		
Natural frequency	1st mode : 0.5000 Hz 2nd mode : 1.4890 Hz 3rd mode : 2.446 Hz		
Modal damping ratio	1st mode : 0.80% 2nd mode : 2.38% 3rd mode : 3.91%		

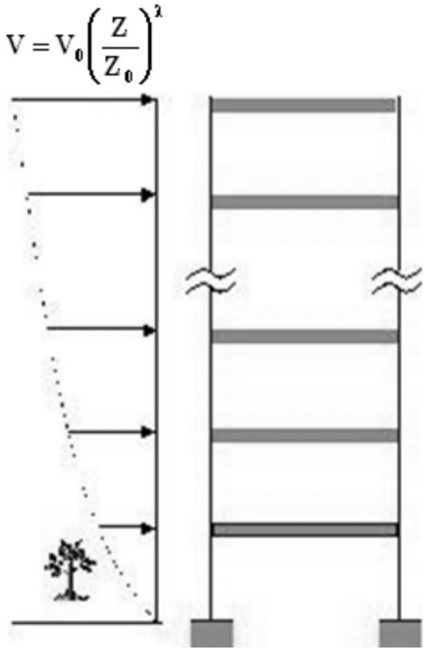

Fig. 1 Wind velocity profile

shown in the following equation and in Fig. 1:

$$V(z) = V_o\left(\frac{z}{z_o}\right)^{\lambda} \tag{23}$$

where z is the height of a story from the ground, z_o is the reference height above the ground, V_o is the velocity at height z_o at a return period of 10 years, and λ is the exponent determining vertical profile of wind velocity. In this study the following values are used for numerical analysis: $z_o = 10$ m, $\lambda = 0.15$, $V_o = 30$ m/sec. For numerical analysis, time history of wind velocity at each story is generated from von Karman's spectrum (Simiu and Scanlan 1996).

3.2. Viscous dampers

Three linear VDs, in which the damper force is proportional to the relative velocity, are installed between the first three inter stories as shown in Fig. 2(a). The equivalent control force provided by the VD can be written as:

$$\begin{aligned} Su(t) &= -c_1\begin{bmatrix}1\\0\\0\\\vdots\\0\end{bmatrix}[\,1\;0\;0\;\cdots\;0\,]\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\vdots\\\dot{x}_{10}\end{pmatrix} \\ &\quad -c_2\begin{bmatrix}-1\\1\\0\\\vdots\\0\end{bmatrix}[\,-1\;1\;0\;\cdots\;0\,]\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\vdots\\\dot{x}_{10}\end{pmatrix} \\ &\quad -c_3\begin{bmatrix}0\\-1\\1\\\vdots\\0\end{bmatrix}[\,0\;-1\;1\;\cdots\;0\,]\begin{pmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\vdots\\\dot{x}_{10}\end{pmatrix} \\ &= -c_1S_1S_1^T\dot{x} - c_2S_2S_2^T\dot{x} - c_3S_3S_3^T\dot{x} \\ &= -S\,diag(c)\,S^T\dot{x} \end{aligned} \tag{24}$$

where c_i is the damping coefficient of the ith VD, $diag(c)$ is the diagonal matrix consisting of c_i, and $\dot{x}_i$ is the velocity of the ith story. Comparing Eq. (24) with Eq. (16), the location matrix, L, and the

Table 2 Modal damping ratios increased by viscous dampers

Analysis methods \ Damping coefficient (c_o)	500	700	1000	1300	1500
Proposed (%)	2.00	2.48	3.18	3.92	4.38
Eigenvalue analysis (%)	2.01	2.49	3.22	3.94	4.42

control force, $u(t)$, are represented as

$$L = S, \qquad u(t) = -diag(c)S^T \dot{x} \tag{25}$$

The control force generated from the *VD* corresponds to the special case of the direct output velocity feedback control in view of the control algorithm, and in this case the feedback gain is $diag(c)$. Considering the mode shape shown in Table 2 and Eq. (24), the generalized control force applied to the *i*th mode can be written as

$$u_i(t) = \frac{\phi_i^T S\, diag(c) S^T}{M_i} \dot{x}(t) \tag{26}$$

where ϕ_i, M_i are the mode shape vector and the modal mass of the *i*th mode, respectively. Substituting Eq. (26) into Eq. (18), the equivalent control energy is calculated through the time history analysis, and the modal EDR of the *i*th mode can be obtained using the Eq. (20) and Eq. (21). Table 2 compares the EDR computed from the proposed method and the modal damping ratio computed from the eigenvalue analysis, which indicates that the EDR of the 1st mode obtained by the proposed approach are very close to those obtained by eigenvalue analysis. For simplicity, the damping coefficient of *VD* in each story is given to be c_o.

3.3. Tuned mass damper (TMD)

The EDR of a primary structure increased by a TMD was proposed as follows by Den Hotdog (1956):

$$\Delta\xi_{eq} = \frac{1}{2}\sqrt{\frac{\mu}{2+\mu}} \tag{27}$$

where it can be noticed that the increase of damping depends on , which is the mass ratio of the TMD and the main structure. The frequency ratio of the main structure and the TMD, γ, and the optimal damping ratio (ξ_{opt}) of TMD are given as

$$\gamma = \frac{1}{1+\mu}, \qquad \xi_{opt} = \frac{1}{2}\sqrt{\frac{3\mu/2}{1+3\mu/2}} \tag{28}$$

The damping ratio of a structure with TMD can be calculated by the eigenvalue analysis. A dynamic system with a TMD coupled with the 1st mode is represented as

$$\begin{bmatrix} \overline{M} & 0 \\ 0 & m_t \end{bmatrix} \begin{pmatrix} \ddot{\eta}_i \\ \ddot{y} \end{pmatrix} + \begin{bmatrix} \overline{C} + c_t\phi_{hi}^2 & -\phi_{hi}c_t \\ -\phi_{hi}c_t & c_t \end{bmatrix} \begin{pmatrix} \dot{\eta}_i \\ \dot{y} \end{pmatrix} + \begin{bmatrix} \overline{K} + \phi_{hi}^2 k_t & -\phi_{hi}k_t \\ -\phi_{hi}k_t & k_t \end{bmatrix} \begin{pmatrix} \eta_i \\ y \end{pmatrix} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} \overline{f} \tag{29}$$

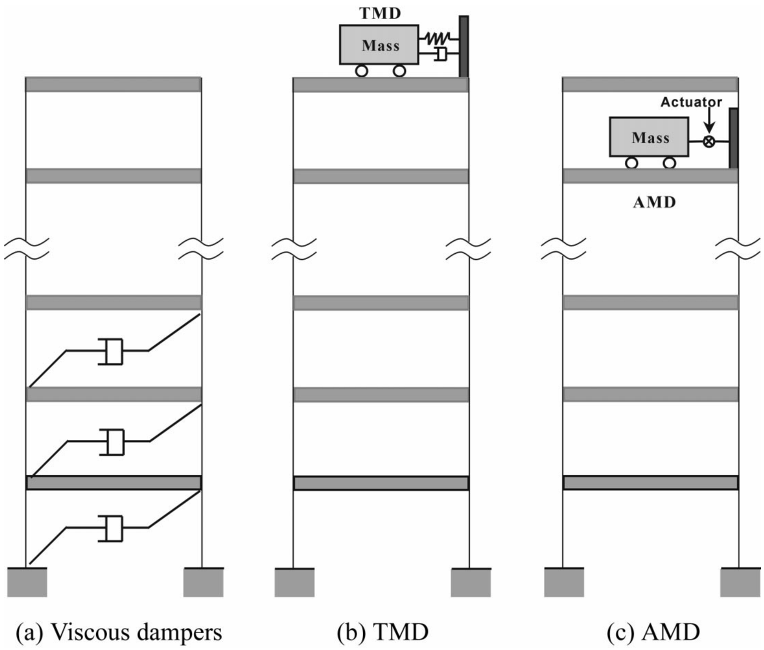

Fig. 2 Model structures with supplemental dampers

where $\bar{M}$, $\bar{C}$, and $\bar{K}$ are the generalized mass, damping, and stiffness of the 1st mode, respectively, as shown in Fig. 2(b), and the *i*th mode shape is normalized by the coefficient corresponding to the top story. Also m_t, c_t, and k_t are the mass, damping, and the stiffness of the TMD, respectively. $\bar{f}$ is the generalized external force including the effect of the other mode, and ϕ_{hi} is the modal coefficient at the *h*th story where the TMD is installed. From an eigenvalue analysis we can get two independent damping ratios of the structure.

Numerical analysis is performed to compare the proposed method with eigenvalue analysis. The TMD is located at the top story as shown in Fig. 2(b), and the frequency ratio and optimal damping ratio obtained from Eq. (28) is used in the analysis.

The equivalent control force induced by a TMD is an inertial force provided by the movement of the mass of the TMD. From Eq. (29) the modal equation of motion for the 1st mode subjected to the equivalent control force can be obtained as follows:

$$\ddot{\eta}_1 + 2\xi_1 w_1 \dot{\eta}_1 + w_1^2 \eta = -\mu \ddot{y} + \bar{f} \tag{30}$$

where μ is the mass ratio of the TMD, and the right-hand-side of the above equation represents the equivalent damping force. The increase of EDR for the 1st mode coupled with the TMD can be predicted from the numerical analysis of the Eq. (30), as mentioned previously.

Table 3 Modal damping ratios increased by TMD

Analysis methods \ Mass ratio (μ)	0.5%	0.8%	1%	1.2%	1.5%
Proposed (%)	2.47	2.75	3.05	3.20	3.71
Eigenvalue analysis (%)	2.04 230	2.44 2.84	2.65 3.16	2.84 3.45	3.09 3.85
Den Hartog (Eq. 27)	3.29	3.95	4.32	4.66	5.11

Table 3 shows the EDR of the structure obtained from various procedures. The inherent damping ratio of the structure, which is assumed to be 0.8% of the critical damping, is also included in the EDR

It can be observed in the table that the damping ratios predicted from the Den Hartog's equation form the upperbound in all mass ratios, mainly because it does not consider the characteristics of the external load. It also can be noticed that the damping ratios obtained from the proposed method are quite close to those from the eigenvalue analysis.

3.4. Active mass driver

The control force supplied by an AMD is dependent on the control algorithm used. In this study the linear quadratic regulator (LQR), which is one of the most popular control algorithms used in building structures, is applied as a control algorithm, and thus the AMD works as a linear damping device. The control gain is found by minimizing the following performance index J:

$$J = \int_0^{\infty} (z^T Q z + u^T R u) dt \tag{31}$$

where Q and R are the weighting matrices associated with the state vector $z(t)$ and control force vector $u(t)$, respectively.

To estimate the damping ratio increased by the AMD, the structure shown in Fig. 2 with the AMD placed on the 9th story is analyzed. The ith modal control force inflicted on the 9th story by the operation of the AMD is as follows:

$$u_i = \phi_{9i} u(t) \tag{32}$$

where ϕ_{9i} is the 9th component of the ith mode shape vector, and $u(t)$ is the control force generated from the AMD, which is expressed as follows:

$$u(t) = -Gz \tag{33}$$

where G is the control gain computed from the weighting matrice Q and R. The matrix Q is

Table 4 Modal damping ratios increased by AMD

Analysis method \ R factor	0.1	0.05	0.03	0.02	0.01
Proposed (%)	3.49	4.85	6.32	7.68	10.82
Eigenvalue analysis (%)	3.51	4.89	6.28	7.66	10.78

obtained from Eq. (5) and R is taken as a variable to regulate the control force. Table 4 compares the equivalent damping ratio obtained from eigenvalue analysis and from the proposed method for various R, where it can be found that the equivalent damping ratios obtained from both methods match quite well.

3.6. Friction dampers

The friction force generated from an ideal Coulomb friction damper can be denoted as follows (Li and Reinhorn 1995):

$$\begin{aligned} F_c &= -C_c\, sign(\dot{x}) \\ &= -C_c \frac{\dot{x}}{|\dot{x}|} \end{aligned} \tag{37}$$

where C_c is the magnitude of the friction force, *sign* (.) is the sign function, and $\dot{x}$ is the relative velocity of the damper. The equation of motion of a SDOF structure with Coulomb-type friction damper, which is shown in Fig. 3, can be denoted as

$$\ddot{x} + 2\xi_o w_o \dot{x} + w_o^2 x = -\frac{C_c}{M} sign(\dot{x}) + \bar{f} \tag{38}$$

The first term in the right-hand-side of Eq. (38) corresponds to the control force contributed from the friction damper, and the equivalent energy dissipated by the control force becomes

$$\begin{aligned} U(t) &= u^T B_2^T Q z + z^T Q B_2 u \\ &= 2\left(\xi_o w_o \frac{C_c}{M} \frac{\dot{x} x}{|\dot{x}|} + \frac{C_c}{M} \frac{\dot{x}^2}{|\dot{x}|} \right) \end{aligned} \tag{39}$$

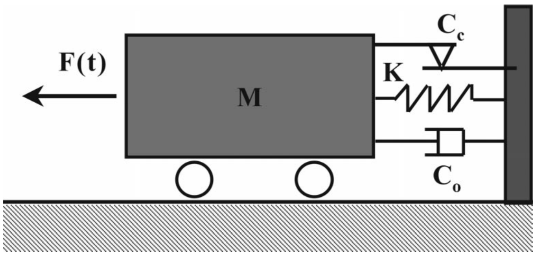

Fig. 3 SDOF system with a friction damper

From the definition of equivalent damping, the damping ratio supplied by the friction damper can be obtained as follows:

$$\xi_{eq} = \frac{1}{2w_o}\frac{2\left(\xi_o w_o \frac{C_c}{M}E\left[\frac{\dot{x}x}{|\dot{x}|}\right] + \frac{C_c}{M}E\left[\frac{\dot{x}^2}{|\dot{x}|}\right]\right)}{w_o^2E[x^2] + \xi_o w_o E[x\dot{x}] + E[\dot{x}^2]} \tag{40}$$

As the displacement and the velocity of a structure are independent of each other and the responses are narrow-band processes, the following relationship holds:

$$E[x\dot{x}] = 0, \quad \sigma_{\dot{x}}^2 = E[\dot{x}^2] = w_o^2 E[x^2] \tag{41}$$

Substituting Eq. (41) into Eq. (40) leads to

$$\xi_{eq} = \frac{1}{2w_o M}\frac{C_c E[|\dot{x}|]}{E[\dot{x}^2]} \tag{42}$$

For a Gaussian probability distribution with the mean velocity of 0 and the standard deviation of $\sigma_{\dot{x}}$, $E[|\dot{x}|]$ can be expressed as

$$E[|\dot{x}|] = 2\int_0^{\infty}\frac{\dot{x}}{\sqrt{2\pi}\,\sigma_{\dot{x}}}\exp\left(-\frac{\dot{x}^2}{2\sigma_{\dot{x}}^2}\right)d\dot{x} = \sqrt{\frac{2}{\pi}}\,\sigma_{\dot{x}} \tag{43}$$

Generally the response of the structure with friction dampers is not Gaussian process because the system is nonlinear. However when the equivalent damping supplied by the added friction damper is small, it can be assumed that the structure behaves similarly to a Gaussian process. By substituting Eq. (43) into Eq. (42), the equivalent damping ratio can be expressed as

$$\xi_{eq}\sigma_{\dot{x}} = \frac{1}{\sqrt{2\pi}}\frac{C_c}{w_o M} \tag{44}$$

Eq. (44) shows that the equivalent damping due to the friction damper is inversely proportional to the response. Table 6 presents the numerical analysis results of the SDOF system shown in Fig. 3 with the dynamic properties shown in Table 5. For the same friction force (C_c) of the damper, the mean and the standard deviation of the velocity are obtained for various magnitudes of the white noise external load. The equivalent damping ratio for each external load is also presented. It can be observed that the responses obtained using the equivalent damping ratio are quite close to those computed directly from the structure with the damper.

Table 5 Modal characteristics of the SDOF system

	Mass	Natural frequency	Damping ratio	Excitation
Values	27.62 kg	0.5075 Hz	0.8%	White noise

Table 6 Response and equivalent damping ratios (C_c = 1.0)

Excitation	No control		With friction dampers			Equivalent damping	
σ_f	$E[\lvert\dot{x}\rvert]$	$\sigma_{\dot{x}}$	$E[\lvert\dot{x}\rvert]$	$\sigma_{\dot{x}}$	ξ_{eq}	$E[\lvert\dot{x}\rvert]$	$\sigma_{\dot{x}}$
0.5232	0.0456	0.0572	0.0075	0.0101	44.85	0.0081	0.0102
0.8356	0.0839	0.1015	0.0181	0.0239	18.95	0.0192	0.0239
1.0429	0.1148	0.1557	0.0310	0.0412	10.99	0.0322	0.0398
1.2542	0.1619	0.1957	0.0411	0.0542	8.36	0.0426	0.0538
1.4655	0.1420	0.1756	0.0541	0.0709	6.39	0.0561	0.0700
1.6766	0.1931	0.2553	0.0804	0.1115	4.06	0.0834	0.1057
1.8837	0.1357	0.1739	0.0765	0.1012	4.48	0.0768	0.0970
2.0902	0.1998	0.2493	0.0967	0.1265	3.58	0.1001	0.1243
2.3026	0.2561	0.3135	0.1114	0.1465	3.09	0.1135	0.1421
2.5051	0.2839	0.3503	0.1712	0.2201	2.06	0.1668	0.2097
2.7189	0.2961	0.3656	0.1810	0.2310	1.96	0.1743	0.2161
2.9282	0.3352	0.4047	0.1949	0.2479	1.83	0.1908	0.2339
3.1355	0.4678	0.5597	0.2778	0.3484	1.30	0.2566	0.3125
4.1745	0.6455	0.7654	0.4429	0.5477	0.83	0.4066	0.4915

4. Conclusions

A procedure for evaluating the equivalent damping ratios of a structure equipped with supplemental damping devices and subjected to a stationary wind load was proposed. Modal-energy of the structure with damping devices was defined as the Lyapunov function, and its derivative was expressed in autoregressive form to obtain the amount of the dissipated energy from the dampers. The equivalent damping ratios were obtained from the Lyapunov function and its derivative, and were used to assess the control effect of various damping devices quantitatively.

Numerical analysis of a structure with linear damping devices such as VDs and AMD using LQR proved that the proposed method can evaluate equivalent damping ratios precisely. The method was also shown to be applicable for estimating equivalent damping ratios of a structure with nonlinear damping devices, for which the eigenvalue analysis cannot be applied.

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