A Behavioral Basis for Best-Shot Public-Good Contests

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Abstract
We examine individual and group equilibrium effort levels in a contest in which two groups compete to win a local public-good prize. The players choose their effort levels simultaneously and independently, the players value the prize differently, and each group's probability of winning is a function of the difference between the two groups' effort levels. We demonstrate that, if a player's equilibrium effort level is positive, he must be the hungriest player in his group. All other players expend no effort. The individual and group equilibrium effort levels depend neither on group size nor the distribution of valuations in each group. Effort depends solely on the best-shot in each group—the valuation of the hungriest player.

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I. Introduction

Collective action coordinates individual effort to lobby the agencies which provide local public goods such as environmental quality and national parks (Olson, 1965; Nitzan, 1994). These collective contests pit one citizenry against another—both sides expending resources to increase the odds their community secures the fixed public good. The odds of success depend on the relative effort expended by both groups, which in turn rests on the technology that coordinates effort, that is, how individual effort adds to total effort. Contest models most commonly represent this technology of effort as the summation of individual effort expended in a community (Katz et al., 1990; Baik, 1993; Riaz et al., 1995). More individual effort across the board, accounting for within-group free riding, leads to more collective effort which ups their odds of success.

This paper demonstrates that when the odds of success are a function of the difference between the effort levels of two collectives, a standard public-good contest with summation technology collapses into a “best-shot” contest. By best-shot, we mean a contest in which collective effort is the effort of the hungriest man or woman—the person who puts the highest value on the fixed public good (see Hirshleifer, 1983). Only the hungriest person in each collective expends effort. The other members of each group free ride completely: they waste not an ounce of effort. This result provides a behavioral motive for a best-shot public-good contest. Rather than assuming the technology ex ante, behavior mimics the best-shot machinery ex post. Regardless of whether he “volunteered” by stepping forward or by all other members stepping back, the summation technology and the difference-form contest-success-function create an incentive for each collective to send forth its hungriest person to win the public good.

One might ask why the difference-form function provides incentive only for the hungriest to fight. Shouldn't an incentive still exist for the second in line to want to help the hungriest, and so on? The answer is no. The gross marginal payoff for the hungriest player is the greatest, while marginal cost is the same for all players in a collective. And
once the hungriest player exerts his optimal effort level, no other player has an incentive to put more effort forward—the existing effort is more than enough. What happens if some players exert effort below the optimal level of the hungriest? The hungriest will add effort up to his optimal level. What happens if some players including the hungriest player exert the optimal effort level of the hungriest? Except for the hungriest, these players have incentive to decrease their effort because their gross marginal payoffs are less than their marginal cost at that effort level.

As such, neither group size nor the distribution of values in the collective affects individual and collective effort levels. And if new blood enters the collective, effort changes only if the newcomer is hungrier than all of the charter members. And while Katz and colleagues (1990) also showed that collective effort is independent of group size, the impetus is quite distinct. Free riding is one-for-one in their model—a dollar increase from a newcomer is offset by a dollar decrease from an original member. But in our model, this trade-off between efforts does not exist: a newcomer either leads or gets out of the way.

II. The Model

Consider a contest in which two groups, 1 and 2, compete with each other for a prize. Group $i$ consists of $m_i$ risk-neutral players who expend effort to win the prize. Let $x_{ik}$ represent the irreversible effort level expended by player $k$ in group $i$ and let $X_i$ represent the effort level expended by all the players in group $i$: $X_i = \sum_{k=1}^{m_i} x_{ik}$. Effort levels are nonnegative and are measured in units commensurate with the prize. Let $P_i(X_1, X_2)$ be the probability that group $i$ wins the prize when the groups' effort levels are $(X_1, X_2)$. Assume the contest success function for group 1 is $P_1(X_1, X_2) = F(D)$ and that for group 2 is $P_2(X_1, X_2) = 1 - F(D)$, where the function $F$ has the properties specified in Assumption 1, and $D = X_1 - X_2$. Each group's probability of winning is a function of the difference between the two groups' effort levels (see Hirshleifer, 1988, 1989, 1991).
**Assumption 1.** Assume $F(0) = 1/2, \ 0 < F(D) < 1$, and $F(-D) = 1 - F(D)$, for all $D$ in $R$, where $R$ denotes the set of all real numbers. Also assume $F'(D) > 0$ for all $D$ in $R$, $F''(D) > 0$ for all $D < 0$, $F''(0) = 0$, and $F''(D) < 0$ for all $D > 0$, where $F'$ and $F''$ denote the first and second derivatives of the function $F$.\(^1\)

Using Assumption 1, we find the following. The groups are symmetric in that groups' probabilities of winning are switched when groups' effort levels are switched. Each group's probability of winning is increasing in its own effort level and is decreasing in the opponent's effort level: $\partial P_i/\partial X_i > 0$ and $\partial P_i/\partial X_j < 0$ for $i \neq j$. (Throughout the paper when we use $i$ and $j$ at the same time, we mean that $i \neq j$.) The marginal effect of group 1's effort level on its own probability of winning increases as its effort level increases when $D$ is negative, and decreases when $D$ is positive: $\partial^2 P_1/\partial X_i^2 > 0$ for all $D < 0$ and $\partial^2 P_1/\partial X_i^2 < 0$ for all $D > 0$. The marginal effect of group 2's effort level on its own probability of winning decreases as its effort level increases when $D$ is negative, and increases when $D$ is positive. Finally, we find $\partial^2 P_1/\partial X_2 \partial X_1 < 0$ and $\partial^2 P_2/\partial X_1 \partial X_2 > 0$ for all $D < 0$, and $\partial^2 P_1/\partial X_2 \partial X_1 > 0$ and $\partial^2 P_2/\partial X_1 \partial X_2 < 0$ for all $D > 0$.

The prize is a public good within each group. Thus, we call it a group-specific public-good prize. Valuations for the prize differ across the players in each group. Let $v_{ik}$ represent the valuation for the prize of player $k$ in group $i$.

**Assumption 2.** Assume $v_{ih-1} > v_{ih} > 0$ for $h = 2, \ldots, m_i$.

Although the players in each group have the same goal of winning the group-specific public-good prize, they choose their effort levels independently. Assume the players in the contest choose their effort levels simultaneously. Let $\pi_{ik}$ represent the expected payoff for player $k$ in group $i$. We have then
\[ \pi_{ik} = v_{ik}P_i(X_1, X_2) - x_{ik}. \]

We assume that all of the above is common knowledge. We employ a Nash equilibrium as the solution concept.

Let \( \bar{x}_{ik} \) denote a best response of player \( k \) in group \( i \), given effort levels of all the other players in the contest. Effort \( \bar{x}_{ik} \) maximizes

\[ \pi_{ik} = v_{ik}P_i(X_1, X_2) - x_{ik} \]

subject to

\[ x_{ik} \geq 0. \]

Effort \( \bar{x}_{ik} \) then satisfies the first-order condition:

\[ v_{ik}(\partial P_i/\partial X_i) - 1 = 0 \quad \text{for } \bar{x}_{ik} > 0, \quad (1) \]

\[ v_{ik}(\partial P_i/\partial X_i) - 1 < 0 \quad \text{for } \bar{x}_{ik} = 0, \quad (2) \]

or

\[ v_{ik}(\partial P_i/\partial X_i) - 1 = 0 \quad \text{for } \bar{x}_{ik} = 0, \quad (3) \]

where \( \partial P_i/\partial X_i = F'(\cdot) \). In the case where \( \bar{x}_{ik} > 0 \), the marginal gross payoff for player \( k \) in group \( i \), \( v_{ik}(\partial P_i/\partial X_i) \), equals his marginal cost, 1, at that effort level. In the case where \( \bar{x}_{ik} = 0 \), his marginal gross payoff does not exceed his marginal cost at that zero effort.

When \( v_{ik}F'(0) > 1 \) holds, either (1) or (2) occurs, depending on the other players' effort levels. When \( v_{ik}F'(0) < 1 \) holds, only (2) occurs. When \( v_{ik}F'(0) = 1 \) holds, either (2) or (3) occurs, depending on the other players' effort levels.

Let an \((m_1 + m_2)\)-tuple vector of effort levels, \((x_{11}^{**}, \ldots, x_{1m_1}^{**}, x_{21}^{**}, \ldots, x_{2m_2}^{**})\), represent a pure-strategy Nash equilibrium of the game. At the Nash equilibrium, each player's
effort level is a best response to the other players' effort levels. This means that, given the other players' equilibrium effort levels, each player's equilibrium effort level satisfies either (1), (2), or (3), in which $\bar{x}_{ik}$ is replaced by $x_{ik}^{**}$.

III. Group-Specific Equilibria

As a preliminary step to characterize a Nash equilibrium of the game, we obtain group-specific equilibria. Given an effort level of group $j$, a group-i-specific equilibrium is an $m_i$-tuple vector of effort levels, one for each player in group $i$, at which each player's effort level is a best response to the other players' effort levels.

We begin by defining group $i$'s player-$k$-best response to group $j$'s effort level.

**Definition 1.** Given group $j$'s effort level, $X_j$, group $i$'s player-$k$-best response, $\bar{X}_i(X_j; v_{ik})$, is defined as an effort level which maximizes

$$v_{ik}P_i(X_1, X_2) - X_i$$

subject to

$$X_i \geq 0.$$

Group $i$'s player-$k$-best response to $X_j$ represents a best response of group $i$ as a whole to $X_j$ when the valuation for the prize of player $k$ in group $i$ is taken into account. More simply, effort $\bar{X}_i(X_j; v_{ik})$ represents a best response of group $i$ to group $j$'s effort level when player $k$ is the only “active” player in group $i$. Effort $\bar{X}_i(X_j; v_{ik})$ then satisfies the first-order condition:

$$v_{ik}(\partial P_i/\partial X_i) - 1 = 0 \quad \text{for } \bar{X}_i(X_j; v_{ik}) > 0,$$

(4)

$$v_{ik}(\partial P_i/\partial X_i) - 1 < 0 \quad \text{for } \bar{X}_i(X_j; v_{ik}) = 0,$$

(5)
or
\[ v_{ik}(\partial P_s/\partial X_j) - 1 = 0 \] for \( X_j; v_{ik} = 0 \), \hspace{1cm} (6)

where \( \partial P_s/\partial X_j = F'(\cdot) \). When \( v_{ik}F'(0) > 1 \) holds, either (4) or (5) occurs, depending on \( X_j \). When \( v_{ik}F'(0) < 1 \) holds, only (5) occurs. When \( v_{ik}F'(0) = 1 \) holds, (6) occurs if \( X_j = 0 \), and (5) occurs otherwise.

We derive group \( i \)'s player-\( k \)-reaction correspondence. It shows group \( i \)'s player-\( k \)-best responses to every possible effort level of group \( j \).

**Lemma 1.** When \( v_{ik}F'(0) > 1 \) holds, group \( i \)'s player-\( k \)-reaction correspondence is: \( \tilde{X}_j(X_j; v_{ik}) = X_j + C_{ik} \) for \( 0 \leq X_j \leq B_{ik} \) and \( \tilde{X}_j(X_j; v_{ik}) = 0 \) for \( X_j \geq B_{ik} \). When \( v_{ik}F'(0) \leq 1 \) holds, group \( i \)'s player-\( k \)-reaction function is: \( \tilde{X}_i(X_j; v_{ik}) = 0 \) for all \( X_j \geq 0 \).

In Lemma 1, \( C_{ik} \) and \( B_{ik} \) are positive constants. \( C_{ik} \) satisfies \( v_{ik}F'(C_{ik}) = 1 \), and \( C_{2k} \) satisfies \( v_{2k}F'(-C_{2k}) = 1 \). Note that \( C_{ik} \) is unique due to Assumption 1. When \( v_{ik}F'(0) > 1 \) holds, drawing group \( i \)'s player-\( k \)-reaction correspondence in the \( X_1,X_2 \)-space, we observe that its “interior” part is parallel to and never coincides with the 45-degree line.

It is straightforward to obtain Lemma 2.

**Lemma 2.** If \( v_{ik}F'(0) > 1 \) holds for \( 2 \leq s \leq m_i \), then \( C_{it-1} > C_{it} \) and \( B_{it-1} > B_{it} \) hold for \( t = 2, \ldots, s \).

Note that \( v_{i1}F'(0) > v_{i2}F'(0) > \cdots > v_{im_i}F'(0) \) holds due to Assumptions 1 and 2. Lemma 2 implies that the interior part of group \( i \)'s reaction correspondence gets longer and farther away from the 45-degree line as a higher valuation is taken into account.

Lemma 3 follows immediately from Lemmas 1 and 2.
Lemma 3. (i) If $v_{it}F'(0) > 1$ holds for $1 \leq s \leq m_i - 1$, then, for $t = 2, \ldots, s + 1$, we have $\bar{X}_i(X_j; v_{it-1}) > \bar{X}_i(X_j; v_{it})$ for $0 \leq X_j < B_{it-1}$, $\bar{X}_i(X_j; v_{it-1}) \geq \bar{X}_i(X_j; v_{it})$ for $X_j = B_{it-1}$, and $\bar{X}_i(X_j; v_{it-1}) = \bar{X}_i(X_j; v_{it}) = 0$ for $X_j > B_{it-1}$. (ii) If $v_{i1}F'(0) \leq 1$ holds, then, for $t = 2, \ldots, m_i$, we have $\bar{X}_i(X_j; v_{it-1}) = \bar{X}_i(X_j; v_{it}) = 0$ for all $X_j \geq 0$.

Since $v_{i1}F'(0) > v_{ih}F'(0)$ holds for $h = 2, \ldots, m_i$, it follows from Lemmas 1 and 3 that, if $v_{i1}F'(0) > 1$ holds, then, for $h = 2, \ldots, m_i$, we have $\bar{X}_i(X_j; v_{i1}) > \bar{X}_i(X_j; v_{ih})$ for $0 \leq X_j < B_{i1}$, $\bar{X}_i(X_j; v_{i1}) \geq \bar{X}_i(X_j; v_{ih})$ for $X_j = B_{i1}$, and $\bar{X}_i(X_j; v_{i1}) = \bar{X}_i(X_j; v_{ih}) = 0$ for $X_j > B_{i1}$. Geometrically, the interior part of group $i$'s player-1-reaction correspondence is longest and farthest away from the 45-degree line.

Now we are prepared to find group-specific equilibria. Let an $m_i$-tuple vector of effort levels, $(x_{i1}^*, \ldots, x_{im_i}^*)$, represent a group-$i$-specific equilibrium given group $j$'s effort level, $X_j$. Lemma 4 shows that group $i$'s effort level at a group-$i$-specific equilibrium given $X_j$ must be equal to group $i$'s player-1-best response to $X_j$: $X_i^* = \bar{X}_i(X_j; v_{i1})$. Lemma 5 constructs group-$i$-specific equilibria given $X_j$.

Lemma 4. Group $i$'s effort level at a group-$i$-specific equilibrium given $X_j$ is neither greater nor less than group $i$'s player-1-best response to $X_j$.

Proof. Consider first the case where $v_{i1}F'(0) < 1$ holds. It follows that $v_{ih}F'(0) < 1$ for $h = 2, \ldots, m_i$. Since $F'(D)$ is maximized at $D = 0$ (see Assumption 1), we obtain: Given $X_j$, for $h = 1, \ldots, m_i$, $v_{ih}F'(X_1 - X_2) - 1 = v_{ih}(\partial P_i/\partial X_i) - 1 < 0$ for any $X_i$. It follows from Lemma 1 that, for any $X_j$, group $i$'s player-1-best response to $X_j$ is zero: $\bar{X}_i(X_j; v_{i1}) = 0$. Suppose that group $i$'s effort level at a group-$i$-specific equilibrium given $X_j$ is positive—greater than $\bar{X}_i(X_j; v_{i1})$. This means there is a player $k$ in group $i$ whose best response to the other players' effort levels is positive. Then, for player $k$, $v_{ik}(\partial P_i/\partial X_i) - 1 = 0$ holds
due to (1). This leads to a contradiction. Therefore, group $i$'s effort level at a group-$i$-specific equilibrium given $X_j$ is not greater than group $i$'s player-1-best response to $X_j$.

Next, when $v_{i1}F'(0) = 1$ holds, the proof is the same as above. The only case that is not covered above is when only player 1 expends effort and his effort level is equal to the given $X_j$. In this case, $v_{i1}(\partial P_i/\partial X_i) - 1 = v_{i1}F'(X_1 - X_2) - 1 = v_{i1}F'(0) - 1 = 0$ holds. But, as Lemma 1 shows, player 1's best response to $X_j$ is zero, and therefore he has an incentive to decrease his effort level.

Now consider the case where $v_{i1}F'(0) > 1$ holds. First, suppose that group $j$'s effort level is given such that $0 \leq X_j < B_{i1}$. It follows from Lemma 1 that $\tilde{X}_i(X_j, v_{i1}) > 0$. If $X_i < \tilde{X}_i(X_j, v_{i1})$ holds, then player 1 in group $i$ has an incentive to increase his effort level. Let $X_i$ be group $i$'s effort level such that $X_i > \tilde{X}_i(X_j, v_{i1})$. Then, by Lemma 3, $X_i > \tilde{X}_i(X_j, v_{i1}) \geq \tilde{X}_i(X_j, v_{ih})$ holds for any $h$, where $1 \leq h \leq m_i$. It follows from this, Assumptions 1 and 2, and (4) that $v_{ih}(\partial P_i/\partial X_i) - 1 = v_{ih}F'(X_1 - X_2) - 1 < 0$ for any $h$. This means that, for any player expending a positive effort level, his effort level is not a best response to the other players' effort levels since his marginal gross payoff is less than his marginal cost (see (1)). The above proves that group $i$'s effort level at a group-$i$-specific equilibrium given $0 \leq X_j < B_{i1}$ is neither greater nor less than group $i$'s player-1-best response to $X_j$.

Second, suppose that $X_j = B_{i1}$. There are two group $i$'s player-1-best responses to $X_j$: one is zero and the other is positive. If $X_i$ is between the two group $i$'s player-1-best responses to $X_j$, then player 1 in group $i$ has an incentive to change his effort level. If $X_i$ is greater than the positive group $i$'s player-1-best response to $X_j$, then, as we show in the previous paragraph, any player expending a positive effort level has an incentive to decrease his effort level. Therefore group $i$'s effort level at a group-$i$-specific equilibrium given $X_j = B_{i1}$ is neither greater nor less than group $i$'s player-1-best response to $X_j$.

Finally, suppose that $X_i > B_{i1}$ and therefore $\tilde{X}_i(X_j, v_{i1}) = 0$. Depending on $X_i$, either $v_{i1}F'(X_1 - X_2) - 1 = v_{i1}(\partial P_i/\partial X_i) - 1 < 0$ or $v_{i1}F'(X_1 - X_2) - 1 = 0$ or $v_{i1}F'(X_1 - X_2) - 1 > 0$ holds. It is straightforward to check that, for any $X_i > 0$, at least
one player has an incentive to change his effort level. It means that no positive effort level is group \( i \)'s effort level at a group-\( i \)-specific equilibrium given \( X_j \).

Lemma 5. Given group \( j \)'s effort level, \( X_j \), a group-\( i \)-specific equilibrium consists of \( x^*_i = \tilde{X}_i(X_j; v_{i1}) \) and \( x^*_h = 0 \) for \( h = 2, \ldots, m_i \).

Proof. Consider first the case where group \( i \)'s player-1-best response to \( X_j \) is zero: \( \tilde{X}_i(X_j; v_{i1}) = 0 \). By Lemma 3, \( \tilde{X}_i(X_j; v_{ik}) = 0 \) holds for any \( k \), where \( 1 \leq k \leq m_i \). Then it follows from (5) and (6) that, if all the players in group \( i \) expend zero effort and thus group \( i \)'s effort level is zero, then \( v_{ik}(\partial P_i/\partial X_i) - 1 \geq 0 \) holds for any \( k \). This implies that, if all the players in group \( i \) expend zero effort, (2) or (3) is satisfied for each player in group \( i \). Therefore, given \( X_j \) and zero effort levels of the other players in group \( i \), zero effort is the best response of each player in group \( i \).

Next, consider the case where \( \tilde{X}_i(X_j; v_{i1}) > 0 \). Using (4) and Assumption 2, we obtain: If group \( i \)'s effort level equals \( \tilde{X}_i(X_j; v_{i1}) \), then \( v_{i1}(\partial P_i/\partial X_i) - 1 = 0 \) and \( v_{ih}(\partial P_i/\partial X_i) - 1 < 0 \) for \( h = 2, \ldots, m_i \) hold. Hence, if player 1 expends \( \tilde{X}_i(X_j; v_{i1}) \) and the rest of the players in group \( i \) choose zero effort levels, (1) is satisfied for player 1, and (2) or (3) is satisfied for each of the rest. Therefore, the proposed effort level of each player in group \( i \) is his best response to \( X_j \) and the proposed effort levels of the other players in group \( i \).

Finally, we show that no other effort levels of the players in group \( i \) constitute a group-\( i \)-specific equilibrium. Suppose on the contrary that a vector of effort levels, \((x_{i1}, \ldots, x_{im_i})\), constitutes a group-\( i \)-specific equilibrium, where \( X_i = \sum_{k=1}^{m_i} x_{ik} = \tilde{X}_i(X_j; v_{i1}) \) and \( x_{it} \neq 0 \) for some \( t \), where \( 2 \leq t \leq m_i \) (see Lemma 4). Then since (1) must be satisfied for player \( t \), at the vector of effort levels \( v_{it}(\partial P_i/\partial X_i) - 1 = 0 \) must hold. Since \( v_{i1} > v_{it} \) due to
Assumption 2, it follows from (4) that at the vector of effort levels \( v_i(\partial P_i/\partial X_i) - 1 < 0 \) must also hold. This leads to a contradiction.

When \( v_{ii}F'(0) > 1 \) holds, given \( X_j = B_{ii} \), there are two group-1's player-1-best responses (see Lemma 1). In this case, there are two group-\( i \)-specific equilibria. Except for that case, given \( X_j \), there is a unique group-\( i \)-specific equilibrium. Lemma 5 says that only the hungriest player—the player with the highest valuation—expends positive effort at a group-specific equilibrium. More correctly, if a player expends positive effort at a group-specific equilibrium, then he must have the highest valuation for the prize in his group. A player whose valuation for the prize is less than somebody else's in his group always expends zero effort and therefore is a free rider. Lemma 5 also says that group \( i \)'s effort level at a group-\( i \)-specific equilibrium given \( X_j \) is equal to a best response of player 1—the hungriest player—in group \( i \) to \( X_j \) when player 1 is the only player in group \( i \) competing against group \( j \).

IV. Nash Equilibrium, Free Riders, and the Best-Shot Contest

We now characterize a Nash equilibrium of the game. Using Lemma 5, we obtain Proposition 1.

**Proposition 1.** If a player in a group expends positive effort at a Nash equilibrium, his valuation for the prize must be the highest one in his group. The equilibrium effort level of the hungriest player in each group is equal to that resulting when only two players, the hungriest player in group 1 and that in group 2, compete to win the prize.

The first part of Proposition 1 implies that a player whose valuation for the prize is less than somebody else's in his group always expends zero effort and free rides. The second part implies that the number of players and the distribution of valuations in each
group do not affect the individual players' and groups' equilibrium effort levels as far as the highest valuation in each group does not change.

It follows from Proposition 1 that, to obtain the equilibrium effort level of the hungriest player in each group and the equilibrium groups' effort levels, we only need to solve a reduced game in which only the hungriest players, one for each group, compete to win the prize. Baik (1998) considers two-player contests with difference-form contest success functions. He shows that a pure-strategy Nash equilibrium at which both players expend positive effort never occurs. He also shows that, if there exists a pure-strategy Nash equilibrium with positive effort, the whole effort is expended by the player with greater valuation. It means that, if there exists a pure-strategy Nash equilibrium in our game, at most one player expends positive effort, and the player, if any, must be the hungriest of all players in the contest. Note that Proposition 1 holds whether our game has a pure-strategy equilibrium or a mixed-strategy one.

V. Conclusion

This paper reveals a behavioral basis for a best-shot public-good contest. Although the technology of coordination sums individual labor into collective effort, people choose to be represented by their hungriest person. The difference-form contest-success-function creates this incentive because the gross marginal payoff for the hungriest player is the greatest, and marginal cost is the same for all players. Once the hungriest player exerts his effort, no other player has an incentive to put more effort forward because the existing effort is more than enough. As such, individual and collective effort levels are independent of group size and the distribution of values. New blood only changes collective effort if he or she is hungrier than all charter members.

We have considered the simultaneous-move framework. Given Lemma 5, it is straightforward to consider the sequential-move framework in which one group moves before the other does. Specifically, consider a sequential-move game in which players in
group 1 choose their effort levels and then, after observing effort levels of players in group 1, players in group 2 choose theirs. This game has always a subgame-perfect equilibrium. We obtain similar results.
Notes

1. Specific functions which satisfy Assumption 1 are $F(D) = 1/(1 + e^{-bD})$ and

$$F(D) = (\sqrt{2\pi} q)^{-1} \int_{-\infty}^{D} \exp(-z^2/2q^2)dz,$$

where $c > 1$, $b > 0$, and $q > 0$.

2. See Baik (1998) for detailed explanations of the derivation.

References


