

Do rent-seeking groups announce their sharing rules?*

by

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Abstract

We study collective rent seeking between two groups in which each group has the option of releasing or not its sharing-rule information. First, we show that the case where both groups release their sharing-rule information never occurs in equilibrium; when the players are unevenly matched, one group releases its sharing-rule information and the other does not. Then, we select the Pareto-superior equilibrium when the players are unevenly matched. We show that, in this selected equilibrium, the underdog releases its sharing-rule information, and the favorite does not; thus the underdog becomes the leader, and the favorite the follower. In the selected equilibrium, total effort level is minimized, and each player's expected payoff is maximized. On the basis of this, we argue that it benefits both the society and rent seekers to let rent-seeking groups decide on releasing their sharing-rule information.

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1. Introduction

Rent seeking can be seen all around us. Firms or coalitions of firms expend resources to influence government officials who have authority to award monopoly rights. Self-interested pressure groups lobby for government subsidies. Foreign producers exert effort to avoid imposing tariffs or quotas on their products. Interest groups or lobbyists contribute to the election of politicians who promise to advance favorable legislation. Research and development (R&D) joint ventures of firms invest resources to invent new products or new production technologies that will create economic rents for specified periods of time under government protection. Individual politicians or political parties campaign hard to win elections.

The literature on rent seeking is enormous and growing. Important papers in this literature include Tullock (1967, 1980), Krueger (1974), Posner (1975), Hillman and Katz (1984), Appelbaum and Katz (1987), Dixit (1987), Hillman and Riley (1989), Hirshleifer (1989), Katz, Nitzan, and Rosenberg (1990), Ellingsen (1991), Nitzan (1991b), Baik and Shogren (1992), Leininger (1993), Che and Gale (1997), Konrad and Schlesinger (1997), Hurley and Shogren (1998), Morgan (2003), and Baye and Hoppe (2003).¹

Recently, many economists have studied collective rent seeking – that is, competition for a rent among groups of players in which the players in each group first decide jointly how to share the rent among themselves if one of them (or the group) wins it, and then all the players in the groups simultaneously and independently choose their effort levels. In this literature, public or private information is *exogenously* assumed regarding sharing rules. Baik and Lee (2007) and Nitzan and Ueda (2007) consider collective rent seeking between groups in which sharing rules are private information – that is, the players in each group expend their effort without observing the sharing rules to which the players in the other groups agreed. All the rest of the literature considers the case where sharing rules are public information – that is, the players in each group expend their effort after observing the sharing rules of the other groups: See, for example, Nitzan (1991a,b), Baik (1994), Lee (1995), Hausken (1995), Davis and Reilly (1999), Baik and Lee (2001), Ueda (2002), and Baik et al. (2006).

However, one may well expect that each group has the *option* of releasing or not its sharing-rule information. Accordingly, the purpose of this paper is to study collective rent seeking between two groups in which the groups first decide independently whether or not to release their sharing-rule information; then, the groups announce their decisions simultaneously before choosing their sharing rules. We examine the groups' decisions on releasing sharing-rule information, their sharing rules, and the effort levels and payoffs of the individual players in equilibrium.

We formally consider the following three-stage game. In the first stage, each group decides and announces whether it will release to the rival group the information about its sharing rule, which will be determined in the second stage. In the second stage, the players in each group jointly choose their sharing rule, and then each group releases the information about its sharing rule if it decided to do so in the first stage. In the third stage, all the players in both groups choose their effort levels simultaneously and independently. At the end of this third stage, the winning *player* is chosen, and the winner shares the prize with the other players in his group according to the sharing rule on which they agreed in the second stage.

Solving the game, we find the following. First, the case where both groups release their sharing-rule information never occurs in equilibrium. Second, the case where neither group releases its sharing-rule information occurs only if the players are evenly matched. Third, when the players are *unevenly* matched, one group releases its sharing-rule information and the other does not. In this case, if the players coordinate to attain the Pareto-superior expected payoffs, the underdog releases its sharing-rule information and the favorite does not.²

This paper is related to Müller and Wärneryd (2001), Stein and Rapoport (2004), Garfinkel (2004), Hausken (2005), and Inderst, Müller, and Wärneryd (2007). Müller and Wärneryd (2001) consider a situation in which managers in a firm jointly produce a surplus, and confront a costly distributional conflict among themselves over the produced surplus. They compare two ownership structures, inside ownership and outside ownership, in several respects. They show that, with outside ownership, less resources may be wasted and the managers have

less incentive to make firm-specific investments. Stein and Rapoport (2004) consider two different contest structures, the between-group model and the semi-finals model. In the between-group model, the groups first compete with one another to win the prize, and then the players in the winning group compete against each other to win the prize. In the semi-finals model, each group first selects the finalist from among its players, and then the finalists – one for each group – compete to win the prize. They show that the semi-finals contest structure tends to generate greater expenditures than the between-group contest structure. Garfinkel (2004) studies endogenous alliance formation and its effect on the severity of conflict in a three-stage model of distributional conflict in which individuals can form alliances in the first stage. Hausken (2005) considers two distinct but related models: the production and conflict model in which each agent allocates his resource between production and fighting, and the rent-seeking model in which each agent uses his resource only for fighting. He compares the production and conflict model and the rent-seeking model in several respects. Inderst, Müller, and Wärneryd (2007) look at influence costs incurred due to distributional conflict in organizations. They find that influence costs may be lower in multidivisional organizations than single-tier organizations.

The paper proceeds as follows. In Section 2, we present the model and set up the game. In Section 3, we analyze the four subgames that start at the second stage of the full game. We obtain the groups' sharing rules, the players' effort levels, and their expected payoffs in each subgame. Section 4 analyzes the first stage of the full game – that is, we examine the groups' decisions on releasing sharing-rule information. In Section 5, we select the Pareto-superior equilibrium in the case where the players are unevenly matched, and discuss the outcomes in the selected equilibrium. In Section 6, we discuss a possible commitment problem. Finally, Section 7 offers our conclusions.

2. The model

Consider a rent-seeking contest (or, in general, a contest) in which players compete by expending irreversible effort to win a rent or a prize. Each player belongs to one of two groups,

1 and 2. Each group consists of n players, where $n \geq 2$, and all the players are risk-neutral. The players' valuations for the prize may differ: Each player in group 1 values the prize at v_1 and each player in group 2 values it at v_2 . The prize will be awarded to one of the *players*.³ Let x_{ik} represent the effort level expended by player k in group i , and let X_i represent the effort level expended by all the players in group i , so that $X_i = \sum_{k=1}^n x_{ik}$. If the total effort level is positive, the probability that player k in group 1 wins the prize is given by $p_{1k} = \gamma x_{1k} / (\gamma X_1 + X_2)$, where $\gamma > 0$, and the probability that player k in group 2 wins is given by $p_{2k} = x_{2k} / (\gamma X_1 + X_2)$.⁴ Note that given the total effort level, the probability that a player wins the prize depends only on his own effort level, not on his group's effort level. The probability of winning for each player is equal to $1/2n$ if all the players expend zero effort. The parameter γ represents abilities of group 1's players in the contest relative to those of group 2's players. For example, if $\gamma > 1$, it means that each player in group 1 has more ability than each player in group 2 – in other words, when they exert the same effort, each player in group 1 has a greater probability of winning than each player in group 2.

The winner "shares" the prize with the other players in his group. If a player in group i wins the prize, the winning player takes $\sigma_i v_i$ and each losing player in the group "takes" $(1 - \sigma_i)v_i/(n - 1)$, where $\sigma_i \geq 1/n$.⁵ We call σ_i the winner's fractional share of group i , which the players in the group agree on before they choose their effort levels.⁶ If $\sigma_i = 1/n$ holds, the players in group i share the prize equally when a player in that group wins it. In the case where $1/n \leq \sigma_i < 1$, the winner takes less than the prize. When the winner's fractional share is equal to unity, the winner takes all the prize. In the case where $\sigma_i > 1$, the winner takes all the prize and further receives "bounties" from the other players in his group. Thus, in this case, the winner earns more than the prize.

Let π_{ik} represent the expected payoff for player k in group i . Then the payoff function for player k in group i is

$$\pi_{ik} = \sigma_i v_i p_{ik} + \{(1 - \sigma_i)v_i/(n - 1)\} \sum_{j \neq k}^n p_{ij} - x_{ik}, \quad (1)$$

where p_{ik} is the probability that player k in group i wins the prize and $\sum_{j \neq k}^n p_{ij}$ is the probability that any one of the other players in group i wins the prize.

We formally consider the following three-stage game. In the first stage, each group decides independently whether it will release to the rival group the information about its sharing rule (or, equivalently, its winner's fractional share), which will be determined in the second stage. The groups announce their decisions simultaneously. In the second stage, the players in each group jointly choose their sharing rule, and then each group releases the information about its sharing rule if it decided to do so in the first stage.⁷ In the third stage, all the players in both groups choose their effort levels simultaneously and independently. At the end of this third stage, the winning player is chosen, and the winner "shares" the prize with the other players in his group according to the sharing rule on which they agreed in the second stage. We assume that there is no transaction cost associated with negotiating an agreement and enforcing compliance. We also assume that all of the above is common knowledge among the players.

3. The four subgames starting at the second stage

We first analyze the subgames that start at the second stage of the full game. There are four such subgames: the (NR, NR) subgame, the (R, NR) subgame, the (NR, R) subgame, and the (R, R) subgame, where NR denotes the action of announcing, in the first stage, that the sharing-rule information will not be released and R the action of announcing that it will be released. The (NR, NR) subgame (also called the no-release subgame) arises when both groups announce that they will not release their sharing-rule information. If group 1 announces that it will release its sharing-rule information but group 2 announces the opposite, then the (R, NR) subgame arises. The (NR, R) subgame arises when group 1 announces that it will not release its sharing-rule information but group 2 announces the opposite. Finally, the (R, R) subgame (also called the bilateral-release subgame) arises when both groups announce that they will release their sharing-rule information.

3.1. The (NR, NR) subgame

In the no-release subgame, the players in each group first choose their sharing rule jointly, and then choose their effort levels simultaneously and independently, without observing the other group's sharing rule or effort levels.⁸ To solve the subgame, we need to find the groups' sharing rules and the players' effort levels which satisfy the following two requirements. First, each player's effort level is optimal given the sharing rule of his own group and given the effort levels of all the other players. That is, each player's effort level is a best response to his group's sharing rule and the effort levels of all the other players. Second, each group's sharing rule is optimal given the effort levels of the players in the other group and given the *subsequent* effort levels of the players in the group.

To obtain such equilibrium actions – the groups' sharing rules and the players' effort levels in equilibrium – we begin by deriving the reaction functions for the players in group 1. Working backward, we first consider the players' decisions on their effort levels. After observing his group's sharing rule or equivalently σ_1 , player k in group 1 seeks to maximize his payoff (1) over his effort level x_{1k} , taking the effort levels of all the other players as given.⁹ We focus on the symmetric equilibrium actions. Thus, let $x_{1k} = x_1$ and $x_{2k} = x_2$ for all k . Then the first-order condition for maximizing (1) reduces to

$$\gamma^2 n^2 x_1^2 + \{2\gamma n^2 x_2 - \gamma^2(\sigma_1 n - 1)v_1\}x_1 + nx_2(nx_2 - \gamma\sigma_1 v_1) = 0.$$

Using this equation, we obtain the following reaction function:

$$x_1(\sigma_1, x_2) = \{\gamma v_1(\sigma_1 n - 1) - 2n^2 x_2 + \sqrt{\gamma^2 v_1^2(\sigma_1 n - 1)^2 + 4\gamma v_1 n^2 x_2}\}/2\gamma n^2. \quad (2)$$

Next, consider the players' decision on their sharing rule. Because the players expend the same effort level, they have the same expected payoff: $\pi_{1k} = \pi_1$ for all k . The players seek to maximize

$$\pi_1(\sigma_1, x_2) = \gamma v_1 x_1(\sigma_1, x_2)/n\{\gamma x_1(\sigma_1, x_2) + x_2\} - x_1(\sigma_1, x_2) \quad (3)$$

with respect to σ_1 , taking group 2's total effort level X_2 , or rather x_2 , as given. Note that we obtain Equation (3) by substituting Equation (2) into Equation (1). From the first-order condition for maximizing Equation (3), we obtain another reaction function of group 1:

$$\sigma_1(x_2) = \{1 + (n - 1)\sqrt{nx_2/\gamma v_1}\}/n. \quad (4)$$

Now consider group 2. After observing his group's sharing rule or equivalently σ_2 , player k in group 2 seeks to maximize his payoff (1) over his effort level x_{2k} , taking the effort levels of all the other players as given. We focus on the symmetric equilibrium actions. Thus the first-order condition for maximizing Equation (1) reduces to

$$n^2x_2^2 + \{2\gamma n^2x_1 - (\sigma_2n - 1)v_2\}x_2 + \gamma nx_1(\gamma nx_1 - \sigma_2v_2) = 0.$$

Using this equation, we obtain the following reaction function:

$$x_2(\sigma_2, x_1) = \{v_2(\sigma_2n - 1) - 2\gamma n^2x_1 + \sqrt{v_2^2(\sigma_2n - 1)^2 + 4\gamma v_2n^2x_1}\}/2n^2. \quad (5)$$

Next, consider the players' decision on their sharing rule. Because the players expend the same effort level, they have the same expected payoff: $\pi_{2k} = \pi_2$ for all k . The players seek to maximize

$$\pi_2(\sigma_2, x_1) = v_2x_2(\sigma_2, x_1)/n\{\gamma x_1 + x_2(\sigma_2, x_1)\} - x_2(\sigma_2, x_1) \quad (6)$$

with respect to σ_2 , taking group 1's total effort level X_1 , or rather x_1 , as given. Note that we obtain Equation (6) by substituting Equation (5) into Equation (1). From the first-order condition for maximizing Equation (6), we obtain another reaction function of group 2:

$$\sigma_2(x_1) = \{1 + (n - 1)\sqrt{\gamma nx_1/v_2}\}/n. \quad (7)$$

We are now ready to obtain the symmetric equilibrium actions, denoted by the $2(n + 1)$ -tuple vector of actions $(\sigma_1^{NR}, x_1^{NR}, \dots, x_1^{NR}, \sigma_2^{NR}, x_2^{NR}, \dots, x_2^{NR})$, by solving the system of four

simultaneous equations, (2), (4), (5), and (7). Substituting Equation (4) into Equation (2), and Equation (7) into Equation (5), we have

$$x_1(x_2) = (\sqrt{\gamma n v_1 x_2} - n x_2) / n \gamma$$

and

$$x_2(x_1) = (\sqrt{\gamma n v_2 x_1} - \gamma n x_1) / n.$$

By solving this pair of simultaneous equations, we obtain the players' equilibrium effort levels, x_1^{NR} and x_2^{NR} . Next, substituting x_2^{NR} into Equation (4), and x_1^{NR} into Equation (7), we obtain the groups' equilibrium sharing rules, σ_1^{NR} and σ_2^{NR} , respectively. Finally, substituting these equilibrium actions into Equations (3) and (6), we obtain the players' equilibrium expected payoffs, π_1^{NR} and π_2^{NR} .

Lemma 1 summarizes the outcomes of the (NR, NR) subgame.

Lemma 1. (a) *In the symmetric equilibrium of the no-release subgame, group 1 chooses $\sigma_1^{NR} = (\gamma v_1 + n v_2) / n(\gamma v_1 + v_2)$, and each player in group 1 expends $x_1^{NR} = \gamma v_1^2 v_2 / n(\gamma v_1 + v_2)^2$. Group 2 chooses $\sigma_2^{NR} = (\gamma n v_1 + v_2) / n(\gamma v_1 + v_2)$, and each player in group 2 expends $x_2^{NR} = \gamma v_1 v_2^2 / n(\gamma v_1 + v_2)^2$. (b) *The expected payoff for each player in group 1 and that for each player in group 2 are $\pi_1^{NR} = \gamma^2 v_1^3 / n(\gamma v_1 + v_2)^2$ and $\pi_2^{NR} = v_2^3 / n(\gamma v_1 + v_2)^2$.**

3.2. The unilateral-release subgames

Consider first the (R, NR) subgame. In this subgame, group 1 releases its sharing-rule information, but group 2 does not. Thus, when choosing their effort levels, the players in both groups know group 1's sharing rule, but only the players in group 2 know group 2's sharing rule.¹⁰

We solve this subgame by viewing it as the following two-stage game. In the first stage, group 1 chooses its sharing rule and announces it publicly. In the second stage, after observing

group 1's sharing rule, groups 1 and 2 play a simultaneous-move game. That is, the players in group 1 choose their effort levels, without observing group 2's sharing rule or effort levels; the players in group 2 choose sequentially their sharing rule and effort levels without observing group 1's effort levels.¹¹

To solve this two-stage game, we work backwards. In the second stage, the players in both groups know group 1's sharing rule, σ_1 . We begin by deriving the reaction functions for the players in group 1. After observing σ_1 , player k in group 1 seeks to maximize his payoff (1) over his effort level x_{1k} , taking the effort levels of all the other players as given. We focus on the symmetric equilibrium actions. Thus, let $x_{1k} = x_1$ and $x_{2k} = x_2$ for all k . Then from the first-order condition for maximizing Equation (1), we obtain the following reaction function:

$$x_1(x_2; \sigma_1) = \{\gamma v_1(\sigma_1 n - 1) - 2n^2 x_2 + \sqrt{\gamma^2 v_1^2 (\sigma_1 n - 1)^2 + 4\gamma v_1 n^2 x_2}\} / 2\gamma n^2. \quad (8)$$

Next, consider group 2. The players in group 2 choose sequentially their sharing rule and effort levels without observing group 1's effort levels. Taking exactly the same steps as in Section 3.1, we obtain the reaction functions of group 2:

$$x_2(\sigma_2, x_1) = \{v_2(\sigma_2 n - 1) - 2\gamma n^2 x_1 + \sqrt{v_2^2 (\sigma_2 n - 1)^2 + 4\gamma v_2 n^2 x_1}\} / 2n^2 \quad (9)$$

and

$$\sigma_2(x_1) = \{1 + (n - 1)\sqrt{\gamma n x_1 / v_2}\} / n. \quad (10)$$

These reaction functions are the same as those in Equations (5) and (7). The reason is that knowing σ_1 does not make any difference because the payoffs to the players in group 2 do not depend directly on σ_1 .

Then, by solving the system of three simultaneous equations, (8), (9), and (10), we obtain

$$\begin{aligned} x_1(\sigma_1) &= \gamma n \sigma_1^2 v_1^2 v_2 / (\gamma v_1 + n v_2)^2, \\ x_2(\sigma_1) &= \gamma \sigma_1 v_1 v_2 \{\gamma(1 - \sigma_1 n)v_1 + n v_2\} / (\gamma v_1 + n v_2)^2, \end{aligned} \quad (11)$$

and

$$\sigma_2(\sigma_1) = [\gamma v_1 \{1 + n(n - 1)\sigma_1\} + n v_2] / n(\gamma v_1 + n v_2).$$

These are the equilibrium effort levels of the players in group 1, those of the players in group 2, and group 2's equilibrium sharing rule, respectively, in the second stage.

Next, consider the first stage in which group 1 chooses its sharing rule. Because the players in group 1 expend the same effort level in the second stage, we have: $\pi_{1k} = \pi_1$ for all k . Having perfect foresight about $\pi_1(\sigma_1)$, the players in group 1 choose their sharing rule which maximizes

$$\pi_1(\sigma_1) = \gamma v_1 x_1(\sigma_1) / n \{ \gamma x_1(\sigma_1) + x_2(\sigma_1) \} - x_1(\sigma_1). \quad (12)$$

Note that we obtain Equation (12) by substituting $x_1(\sigma_1)$ and $x_2(\sigma_1)$ in Equation (11) into Equation (1). From the first-order condition for maximizing Equation (12) with respect to σ_1 , we obtain group 1's equilibrium sharing rule, σ_1^{1R} .

Now, substituting σ_1^{1R} into $x_1(\sigma_1)$, $x_2(\sigma_1)$, and $\sigma_2(\sigma_1)$ in Equation (11), we obtain the players' equilibrium effort levels, x_1^{1R} and x_2^{1R} , and group 2's equilibrium sharing rule, σ_2^{1R} , respectively. Next, using these equilibrium actions, we obtain the players' equilibrium expected payoffs, π_1^{1R} and π_2^{1R} .

Lemma 2 summarizes the outcomes of the (R, NR) subgame.

Lemma 2. (a) *In the symmetric equilibrium of the (R, NR) subgame, group 1 chooses $\sigma_1^{1R} = (\gamma v_1 + n v_2) / 2 n v_2$, and each player in group 1 expends $x_1^{1R} = \gamma v_1^2 / 4 n v_2$. Group 2 chooses $\sigma_2^{1R} = \{ \gamma (n - 1) v_1 + 2 v_2 \} / 2 n v_2$, and each player in group 2 expends $x_2^{1R} = \gamma v_1 (2 v_2 - \gamma v_1) / 4 n v_2$. (b) *The expected payoff for each player in group 1 and that for each player in group 2 are $\pi_1^{1R} = \gamma v_1^2 / 4 n v_2$ and $\pi_2^{1R} = (2 v_2 - \gamma v_1)^2 / 4 n v_2$.**

Next, consider the other unilateral-release subgame, the (NR, R) subgame. In this subgame, group 2 releases its sharing-rule information, but group 1 does not. Thus, when choosing their effort levels, the players in both groups know group 2's sharing rule, but only the players in group 1 know group 1's sharing rule.

We solve this subgame by viewing it as the following two-stage game. In the first stage, group 2 chooses its sharing rule and announces it publicly. In the second stage, after observing group 2's sharing rule, groups 1 and 2 play a simultaneous-move game. That is, the players in group 1 choose sequentially their sharing rule and effort levels without observing group 2's effort levels; the players in group 2 choose their effort levels, without observing group 1's sharing rule or effort levels.

Because the analysis is similar to that for the (R, NR) subgame, we only report the results. Lemma 3 summarizes the outcomes of the (NR, R) subgame.

Lemma 3. (a) *In the symmetric equilibrium of the (NR, R) subgame, group 1 chooses $\sigma_1^{2R} = \{2\gamma v_1 + (n-1)v_2\}/2\gamma n v_1$, and each player in group 1 expends $x_1^{2R} = v_2(2\gamma v_1 - v_2)/4\gamma^2 n v_1$. Group 2 chooses $\sigma_2^{2R} = (\gamma n v_1 + v_2)/2\gamma n v_1$, and each player in group 2 expends $x_2^{2R} = v_2^2/4\gamma n v_1$. (b) *The expected payoff for each player in group 1 and that for each player in group 2 are $\pi_1^{2R} = (2\gamma v_1 - v_2)^2/4\gamma^2 n v_1$ and $\pi_2^{2R} = v_2^2/4\gamma n v_1$.**

3.3. The (R, R) subgame

The bilateral-release subgame has two stages. In the first stage, the players in each group jointly choose their sharing rule and announce it publicly. In the second stage, after observing the sharing rules, the players in both groups choose their effort levels simultaneously and independently.

To solve for a subgame-perfect equilibrium of this subgame, we work backwards. In the second stage, the players in both groups know the groups' sharing rules, σ_1 and σ_2 . We begin by deriving the reaction functions for the players in group 1. Player k in group 1 seeks to maximize his payoff (1) over his effort level x_{1k} , taking the effort levels of all the other players as given. We focus on the symmetric equilibrium. Thus, let $x_{1k} = x_1$ and $x_{2k} = x_2$ for all k . Then from the first-order condition for maximizing Equation (1), we obtain the following reaction function:

$$x_1(x_2; \sigma_1, \sigma_2) = \{\gamma v_1(\sigma_1 n - 1) - 2n^2 x_2 + \sqrt{\gamma^2 v_1^2(\sigma_1 n - 1)^2 + 4\gamma v_1 n^2 x_2}\}/2\gamma n^2.$$

Similarly, the reaction function for each player in group 2 is

$$x_2(x_1; \sigma_1, \sigma_2) = \{v_2(\sigma_2 n - 1) - 2\gamma n^2 x_1 + \sqrt{v_2^2(\sigma_2 n - 1)^2 + 4\gamma v_2 n^2 x_1}\}/2n^2.$$

Using these reaction functions, we obtain the symmetric Nash equilibrium in the second stage of the bilateral-release subgame:¹²

$$x_1(\sigma_1, \sigma_2) = v_1 v_2 \{(\sigma_1 + \sigma_2)n - 1\} \{\gamma \sigma_1 n v_1 - (\sigma_2 n - 1)v_2\} / (\gamma n v_1 + n v_2)^2$$

and (13)

$$x_2(\sigma_1, \sigma_2) = \gamma v_1 v_2 \{(\sigma_1 + \sigma_2)n - 1\} \{\sigma_2 n v_2 - \gamma(\sigma_1 n - 1)v_1\} / (\gamma n v_1 + n v_2)^2.$$

Let $\pi_i(\sigma_1, \sigma_2)$ be the expected payoff of each player in group i at the symmetric Nash equilibrium of the second stage. Substituting $x_1(\sigma_1, \sigma_2)$ and $x_2(\sigma_1, \sigma_2)$ into Equation (1), we obtain

$$\pi_1(\sigma_1, \sigma_2) = \gamma v_1 x_1(\sigma_1, \sigma_2) / n \{\gamma x_1(\sigma_1, \sigma_2) + x_2(\sigma_1, \sigma_2)\} - x_1(\sigma_1, \sigma_2) \quad (14)$$

and

$$\pi_2(\sigma_1, \sigma_2) = v_2 x_2(\sigma_1, \sigma_2) / n \{\gamma x_1(\sigma_1, \sigma_2) + x_2(\sigma_1, \sigma_2)\} - x_2(\sigma_1, \sigma_2).$$

Next, consider the first stage in which both groups choose their sharing rules. In this stage, each player has perfect foresight about both $\pi_1(\sigma_1, \sigma_2)$ and $\pi_2(\sigma_1, \sigma_2)$. Given the other group's sharing rule, the players in group i choose their sharing rule that maximizes $\pi_i(\sigma_1, \sigma_2)$. From the first-order condition for maximizing $\pi_i(\sigma_1, \sigma_2)$ for $i = 1, 2$, we obtain the following reaction functions, $\sigma_1(\sigma_2)$ for group 1 and $\sigma_2(\sigma_1)$ for group 2, respectively:

$$\sigma_1(\sigma_2) = \{(v_2 - \gamma v_1)/2\gamma v_1\}\sigma_2 + \{\gamma^2 n v_1^2 + \gamma(n+1)v_1 v_2 - v_2^2\}/2\gamma n v_1 v_2 \quad (15)$$

and

$$\sigma_2(\sigma_1) = \{(\gamma v_1 - v_2)/2v_2\}\sigma_1 + \{n v_2^2 + \gamma(n+1)v_1 v_2 - \gamma^2 v_1^2\}/2\gamma n v_1 v_2.$$

Using these two reaction functions, we obtain the groups' sharing rules, σ_1^{BR} and σ_2^{BR} , which are specified in the subgame-perfect equilibrium of the bilateral-release subgame:

$$\sigma_1^{BR} = \{\gamma^2(2n+1)v_1^2 - \gamma(n+1)v_1v_2 + nv_2^2\}/\gamma nv_1(\gamma v_1 + v_2)$$

and

$$\sigma_2^{BR} = \{\gamma^2 nv_1^2 - \gamma(n+1)v_1v_2 + (2n+1)v_2^2\}/nv_2(\gamma v_1 + v_2).$$

Now, substituting σ_1^{BR} and σ_2^{BR} into $x_1(\sigma_1, \sigma_2)$ and $x_2(\sigma_1, \sigma_2)$ in Equation (13), we obtain the players' equilibrium effort levels, x_1^{BR} and x_2^{BR} , respectively. Next, using these equilibrium actions, we obtain the players' equilibrium expected payoffs, π_1^{BR} and π_2^{BR} .

Let $Q \equiv \gamma^2 nv_1^2 + nv_2^2 - \gamma v_1 v_2$, $G \equiv \gamma(n+1)v_1 - nv_2$, $D \equiv n^2(\gamma v_1 + v_2)^2$, and $H \equiv (n+1)v_2 - \gamma nv_1$. Lemma 4 summarizes the outcomes of the (R, R) subgame.

Lemma 4. (a) *In the symmetric equilibrium of the (R, R) subgame, group 1 chooses σ_1^{BR} , and each player in group 1 expends $x_1^{BR} = Q \times G/\gamma D$. Group 2 chooses σ_2^{BR} , and each player in group 2 expends $x_2^{BR} = Q \times H/D$. (b) *The expected payoff for each player in group 1 and that for each player in group 2 are $\pi_1^{BR} = v_2 \times G^2/\gamma D$ and $\pi_2^{BR} = \gamma v_1 \times H^2/D$.**

4. Groups' decisions on releasing sharing-rule information

Now consider the first stage of the full game. In this stage, each group first decides independently whether or not to release its sharing-rule information to the rival group, and then both groups announce their decisions simultaneously. In short, each group chooses one of the following two actions: announcing that it will not release its sharing-rule information or announcing that it will release the information. Recall that we denote the former action by NR and the latter one by R . We then have four possible combinations of the groups' actions: (NR, NR) , (R, NR) , (NR, R) , and (R, R) . For example, if group 1 chooses NR and group 2 chooses R in the first stage, then the combination (NR, R) arises. Table 1 shows the expected payoffs of the players for the four possible combinations. The combination (NR, NR) leads to the no-release subgame analyzed in Section 3.1, so that the expected payoff for each player in group 1 is π_1^{NR} , and that for each player in group 2 is π_2^{NR} (see Lemma 1). Similarly, we have π_1^{1R} and π_2^{1R} for the

combination (R, NR) , π_1^{2R} and π_2^{2R} for the combination (NR, R) , and π_1^{BR} and π_2^{BR} for the combination (R, R) , which come from Lemmas 2, 3, and 4, respectively.

Which combinations occur in the equilibria of the full game? Looking at Table 1 that illustrates the strategic interaction between the groups in the first stage, we have the following. If $\pi_1^{NR} \geq \pi_1^{1R}$ and $\pi_2^{NR} \geq \pi_2^{2R}$, then the combination (NR, NR) occurs in equilibrium; if $\pi_1^{1R} \geq \pi_1^{NR}$ and $\pi_2^{1R} \geq \pi_2^{BR}$, then the combination (R, NR) arises; if $\pi_1^{2R} \geq \pi_1^{BR}$ and $\pi_2^{2R} \geq \pi_2^{NR}$, then the combination (NR, R) arises; if $\pi_1^{BR} \geq \pi_1^{2R}$ and $\pi_2^{BR} \geq \pi_2^{1R}$, then the combination (R, R) occurs in equilibrium. Using these statements together with Lemmas 1 through 4, we obtain Proposition 1. For concise exposition, from this point on, we let $v_1 \equiv \alpha v_2$, where $\alpha > 0$.¹³

Proposition 1. (a) *If $\alpha\gamma = 1$, then the combinations, (R, NR) , (NR, R) , and (NR, NR) , occur in the equilibria of the full game. (b) *If $\alpha\gamma \neq 1$, then the combinations, (R, NR) and (NR, R) , occur in the equilibria of the full game.**

When $\alpha\gamma = 1$, all the players in the contest have the same "composite strength" – strength determined by their valuations for the prize as well as their relative abilities. In this case, if they were to compete *individually* to win the prize by exerting effort simultaneously, they would have the same probability of winning in equilibrium. When $\alpha\gamma = 1$, we have $(\pi_1^{1R}, \pi_2^{1R}) = (\pi_1^{2R}, \pi_2^{2R}) = (\pi_1^{NR}, \pi_2^{NR}) = (\alpha v_2/4n, v_2/4n)$ and $(\pi_1^{BR}, \pi_2^{BR}) = (\alpha v_2/4n^2, v_2/4n^2)$. This implies that, given group 2's action NR , each player in group 1 is indifferent between NR and R because $\pi_1^{NR} = \pi_1^{1R}$; given group 1's action NR , each player in group 2 is also indifferent between NR and R because $\pi_2^{NR} = \pi_2^{2R}$; furthermore, the three combinations – (R, NR) , (NR, R) , and (NR, NR) – lead to the same pair of expected payoffs.

Proposition 1 immediately implies that the combination (R, R) – which leads to the Pareto-inferior pair of expected payoffs, (π_1^{BR}, π_2^{BR}) – never occurs in equilibrium.¹⁴ This result is very surprising because all the papers except Baik and Lee (2007) that study collective rent seeking assume that sharing rules are public information. Moreover, it is surprising because this

type of game tends to yield (weakly) dominant actions for the "players" in the first stage which lead to a Pareto-inferior equilibrium.¹⁵ The result can be explained as follows. Given the rival group's action R , group i has two choices, R or NR . If the group chooses R , then both groups will announce their sharing rules. In this case, each group will choose a large winner's fractional share to gain strategic advantage against its rival group in the effort-expending stage. The large winner's fractional shares in turn will motivate the players to expend large effort levels, which will result in significantly small expected payoffs to the players in group i . On the other hand, if group i chooses NR , then only the rival group will announce its sharing rule exercising strategic leadership. In this case, sizing up the rival group's sharing rule, group i will choose a sharing rule with which it can avoid a big fight against the rival group. Consequently, the players will expend moderate effort levels, which will result in sizable expected payoffs to the players in group i . Hence, the players in group i choose NR instead of choosing R .

Proposition 1 says that the equilibrium involving (R, NR) and the equilibrium involving (NR, R) always occur regardless of the value of $\alpha\gamma$. That is, given the rival group's action R , group i has no incentive to deviate from its action NR ; given the rival group's action NR , group i has no incentive to deviate from its action R . The former statement is supported by the explanations in the preceding paragraph. The latter is supported by the following intuitive explanation. With the rival group's action NR and its own action R , group i enjoys a first-mover advantage by announcing its sharing rule before the rival group chooses its sharing rule. However, if group i chooses NR instead of R , it loses the strategic leadership and plays the simultaneous-move game with sequential moves against the rival group, which results in smaller or equal expected payoffs to the players in group i . Thus, group i has no incentive to deviate from its action R .

Another interesting result is that, in equilibrium, there exist the strategical leader and the strategical follower that are determined *endogenously*. Indeed, in the equilibria – except the one involving the combination (NR, NR) – one group chooses R and the other chooses NR ; the former group becomes the leader and the latter one becomes the follower.

5. The underdog becomes the strategical leader

Now a natural and interesting question is: Which group chooses R and becomes the leader in equilibrium? At a first glance of Proposition 1, this question seems to make no sense because the equilibrium involving (R, NR) and the equilibrium involving (NR, R) always exist together. However, the question does make sense because we can narrow the equilibrium set. Below we narrow the equilibrium set in the case where $\alpha\gamma \neq 1$. Using Lemmas 2 and 3, we obtain Lemma 5.

Lemma 5. (a) *If $\alpha\gamma > 1$, then the expected payoffs of the players are greater in the equilibrium involving (NR, R) than in the equilibrium involving (R, NR) : $\pi_i^{2R} > \pi_i^{1R}$ for $i = 1, 2$.* (b) *If $\alpha\gamma < 1$, then the expected payoffs of the players are greater in the equilibrium involving (R, NR) than in the equilibrium involving (NR, R) : $\pi_i^{1R} > \pi_i^{2R}$ for $i = 1, 2$.*

Following Dixit (1987), we call group 1 the favorite and group 2 the underdog, when $\alpha\gamma > 1$; we call group 1 the underdog and group 2 the favorite, when $\alpha\gamma < 1$.

Lemma 5 says that, if $\alpha\gamma > 1$, then the combination (NR, R) leads to a Pareto-superior pair of expected payoffs, compared with the combination (R, NR) ; if $\alpha\gamma < 1$, then the opposite holds true. This implies that the expected payoffs of the players are greater in the equilibrium in which the underdog chooses R and the favorite chooses NR – thus, the underdog becomes the strategical leader. How do we explain this? Consider, for example, the case where $\alpha\gamma > 1$ and thus group 2 is the underdog. In this case, we have $0.5 < \sigma_1^{2R} < \sigma_1^{1R} < 1$ and $0.5 < \sigma_2^{2R} < \sigma_2^{1R} < 1$ (see Lemmas 2 and 3). That is, both groups choose smaller winner's fractional shares in the equilibrium involving (NR, R) – in which the underdog is the leader – than in the equilibrium involving (R, NR) . The smaller winner's fractional shares in turn cause the players to expend smaller effort levels, which result in larger expected payoffs to the players, compared with the equilibrium involving (R, NR) .¹⁶ The remaining question is then: Why do the groups choose smaller winner's fractional shares when the underdog is the leader?

We answer this question as follows. When the underdog is the leader, the underdog restrains itself to avoid stiff competition against the strong rival. This in turn allows the favorite to ease up and respond efficiently. On the other hand, when the favorite is the leader, it preempts by choosing a large winner's fractional share. In response to this preemptive behavior, the underdog follows suit – facing aggressive players in the rival group, the players in the underdog group must make themselves aggressive by choosing a large winner's fractional share.

Now, using Lemma 5, we can narrow the equilibrium set. Let us assume that the groups can coordinate each other in choosing their actions in the first stage. Then we expect that the groups, or rather the players, will end up with the Pareto-superior expected payoffs. This means that, if $\alpha\gamma > 1$, then group 1 chooses *NR* and group 2 chooses *R*; if $\alpha\gamma < 1$, then group 1 chooses *R* and group 2 chooses *NR* in the first stage. Proposition 2 highlights this result.

Proposition 2. *If $\alpha\gamma \neq 1$, then the underdog chooses *R* – that is, the underdog announces that it will release its sharing-rule information – and the favorite chooses *NR* in the first stage. Thus, the underdog becomes the strategical leader and the favorite becomes the strategical follower.*¹⁷

Table 2 presents the outcomes of the contest *in the selected equilibrium*. It says that the favorite chooses a smaller winner's fractional share than the underdog.¹⁸ This happens because the favorite eases up, possessing a competitive advantage over the underdog, whereas the players in the underdog group motivate themselves by choosing a more "selfish" sharing rule to overcome their competitive disadvantage. Table 2 also says that the equilibrium winner's fractional shares are less than unity. This means that, if a player in a group wins the prize, the winner helps the losers in his group. Finally, Table 2 says that each player in the favorite group has a greater probability of winning than each player in the underdog group.¹⁹ Note that, in Table 2, p_i^{2R} represents the probability of winning for each player in group *i* in the equilibrium

involving (NR, R) ; and p_i^{1R} represents the probability of winning for each player in group i in the equilibrium involving (R, NR) .

Of great interest is to compare the total effort level and the expected payoffs in the selected equilibrium with those obtained in the other three subgames – which start at the second stage of the full game – that are not specified in the selected equilibrium. Comparing the total effort level in the selected equilibrium with that obtained in the (NR, NR) subgame, and with that obtained in the (R, R) subgame, we find that the players expend less effort in the selected equilibrium than in each of the two subgames: In terms of symbols, if $\alpha\gamma > 1$, then we have $X_1^{2R} + X_2^{2R} < X_1^{NR} + X_2^{NR}$ and $X_1^{2R} + X_2^{2R} < X_1^{BR} + X_2^{BR}$; if $\alpha\gamma < 1$, then we have $X_1^{1R} + X_2^{1R} < X_1^{NR} + X_2^{NR}$ and $X_1^{1R} + X_2^{1R} < X_1^{BR} + X_2^{BR}$.²⁰ This together with footnote 16 demonstrates that total effort level (or the extent of rent dissipation) is minimized in the selected equilibrium. In other words, the social costs associated with collective rent seeking is minimized in the selected equilibrium.²¹ Next, comparing the expected payoffs in the selected equilibrium with those obtained in the (NR, NR) subgame, and with those obtained in the (R, R) subgame, we find that the expected payoff for each player is greater in the selected equilibrium than in each of the two subgames. More specifically, if $\alpha\gamma > 1$, then we have $\pi_i^{BR} < \pi_i^{NR} < \pi_i^{2R}$ for $i = 1, 2$; if $\alpha\gamma < 1$, then we have $\pi_i^{BR} < \pi_i^{NR} < \pi_i^{1R}$ for $i = 1, 2$. This together with Lemma 5 demonstrates that the expected payoff for each player is maximized in the selected equilibrium. To sum up, in the selected equilibrium, the extent of rent dissipation is minimized, and each player's expected payoff is maximized, compared with the other three subgames that are not specified in the selected equilibrium.²² On the basis of this, we argue that it benefits both the society and rent seekers to let rent-seeking groups *decide freely* on releasing their sharing-rule information. Furthermore, we argue that, to reduce the extent of rent dissipation, policies or regulations or institutions which *require* rent-seeking groups to release or hide their sharing-rule information should not be enacted or established; but those that facilitate rent seekers or rent-seeking groups to coordinate each other in choosing their actions and those that facilitate them to commit to their chosen actions may be established.

6. A possible commitment problem

So far we have abstracted away from the possibility that a group reneges on its first-stage decision on releasing sharing-rule information. However, a group may renege on its first-stage decision if it *can* do so. For example, recall from Section 5 that, in the selected equilibrium, the underdog chooses R and the favorite chooses NR in the first stage – that is, the favorite announces in the first stage that it will not release its sharing-rule information. However, after observing the underdog's equilibrium sharing rule, the favorite may publicly announce – if it can do – its "aggressive" sharing rule, which is the best response to the underdog's "equilibrium" sharing rule. Indeed, in a specific example below, the favorite (or group 1) has an incentive to announce its "aggressive" sharing rule.

Consider the case where $\alpha\gamma > 1$. In this case, group 1 chooses NR and group 2 chooses R in the selected equilibrium – that is, the (NR, R) subgame occurs in the selected equilibrium. Then, using Lemma 3 and that $v_1 \equiv \alpha v_2$, we obtain

$$\begin{aligned}\sigma_1^{2R} &= (2\alpha\gamma + n - 1)/2\alpha\gamma n, \\ \sigma_2^{2R} &= (\alpha\gamma n + 1)/2\alpha\gamma n,\end{aligned}\tag{16}$$

and

$$\pi_1^{2R} = v_2(2\alpha\gamma - 1)^2/4\alpha\gamma^2 n,$$

where σ_1^{2R} is the equilibrium sharing rule (or, equivalently, the equilibrium winner's fractional share) of group 1, σ_2^{2R} is that of group 2, and π_1^{2R} is the expected payoff for each player in group 1 in the selected equilibrium.

Suppose now that group 1 *can and does renege* on its first-stage decision on releasing sharing-rule information. Specifically, suppose that, after observing group 2's equilibrium sharing rule σ_2^{2R} in Equation (16), the players in group 1 announce their sharing rule publicly before choosing their effort levels. Then, group 1 chooses and announces its "aggressive" sharing rule σ_1^{d2R} instead of choosing and announcing its "equilibrium" sharing rule σ_1^{2R} in Equation (16), where $\sigma_1^{d2R} \equiv \sigma_1(\sigma_2^{2R}) = \sigma_1^{2R} + \{2\alpha^3\gamma^3 n + (n - 2)\alpha^2\gamma^2 - (n + 1)\alpha\gamma + 1\}/$

$4\alpha^2\gamma^2n$. Note that we obtain σ_1^{d2R} by substituting σ_2^{2R} in Equation (16) into group 1's reaction function, $\sigma_1(\sigma_2)$ in Equation (15), in the (R, R) subgame. Note also that σ_1^{d2R} is greater than σ_1^{2R} in Equation (16). Next, substituting σ_1^{d2R} and σ_2^{2R} into $x_1(\sigma_1, \sigma_2)$ and $x_2(\sigma_1, \sigma_2)$ in Equation (13), then substituting $x_1(\sigma_1^{d2R}, \sigma_2^{2R})$ and $x_2(\sigma_1^{d2R}, \sigma_2^{2R})$ into Equation (14), and using that $v_1 \equiv \alpha v_2$, we obtain the expected payoff for each player in group 1 from the "deviation":

$$\pi_1^{d2R} \equiv \pi_1(\sigma_1^{d2R}, \sigma_2^{2R}) = v_2(2\alpha\gamma - 1)^2(\alpha\gamma n + 1)^2/16\alpha^2\gamma^3n^2. \quad (17)$$

Now, comparing the expected payoffs, π_1^{2R} in Equation (16) and π_1^{d2R} in Equation (17), we obtain that $\pi_1^{2R} < \pi_1^{d2R}$. This implies that group 1, or rather each player in group 1, would be better off by announcing its "aggressive" sharing rule rather than by keeping its first-stage "promise." In other words, it implies that group 1 reneges on its first-stage action if it can do so.

In the example above, if group 1 is not committed to its first-stage action NR , then group 2 believes that group 1 will announce its sharing rule rather than keep its first-stage "promise." This leads to the outcomes of the (R, R) subgame, not to those of the (NR, R) subgame, and thus the players in both groups are worse off. In short, if group 1 cannot commit to its first-stage action NR , then a commitment problem arises.²³

Clearly, both groups prefer group 1 to be committed to its first-stage action NR . An immediate, natural question is then: How is group 1 committed to that action? Or, in general, how are rent-seeking groups committed to such actions or decisions? We can think of several commitment devices or ways of their being committed. First, the contest organizer, if any, or the person who has authority to select the winner can simply require the groups to keep their "promises." Second, the contest organizer, the decision-maker, or the rent-seeking groups can create institutions or make rules to solve the commitment problem. Third, the rent-seeking groups themselves can have incentives to maintain their reputations for keeping their "promises." Fourth, culture can be a commitment device. For example, feelings of guilt can provide psychological incentives for the groups to keep their "promises." Finally, even though rent-seeking groups are not committed to their earlier actions or decisions, a group may not wish to

start a "war" which will devastate itself as well as its rival groups. In the specific example above, if group 1 *announces* its "aggressive" sharing rule after observing group 2's equilibrium sharing rule, then group 2 may update and re-announce its sharing rule; then, group 1 may update and re-announce its sharing rule; and so on. This leads to lower payoffs for the players in both groups, as compared with the case where group 1 keeps its first-stage "promise."

7. Conclusions

We considered collective rent seeking between two groups in which each group has the option of releasing or not its sharing-rule information. More specifically, we considered the following three-stage game. In the first stage, each group decides and announces whether it will release to the rival group the information about its sharing rule, which will be determined in the second stage. In the second stage, the players in each group jointly choose their sharing rule, and then each group releases the information about its sharing rule if it decided to do so in the first stage. In the third stage, all the players in both groups choose their effort levels simultaneously and independently.

In Section 4, we demonstrated that the case where both groups release their sharing-rule information never occurs in equilibrium; the case where neither group releases its sharing-rule information occurs only if the players are evenly matched. This result is very surprising because almost all the papers in the literature on collective rent seeking assume that sharing rules are public information. We also demonstrated that, when the players are unevenly matched, one group releases its sharing-rule information and the other does not. Because the group that releases its sharing-rule information assumes the leadership role, the former group becomes the strategic leader and the latter becomes the strategic follower – the roles are determined endogenously.

In Section 5, we first showed that the expected payoffs of the players are greater in the equilibrium in which the underdog – the group with "weaker" players – releases its sharing-rule information and the favorite does not, compared with the equilibrium in which the favorite

releases its sharing-rule information and the underdog does not. Then, assuming that the players coordinate to attain the Pareto-superior expected payoffs, we selected an equilibrium that is Pareto superior to the other when the players are unevenly matched. In this selected equilibrium, the underdog releases its sharing-rule information, and the favorite does not; thus the underdog becomes the strategical leader, and the favorite becomes the strategical follower. In the selected equilibrium, total effort level (or the extent of rent dissipation) is minimized, and each player's expected payoff is maximized, compared with the other three subgames that are not specified in the selected equilibrium. On the basis of this, we argue that it benefits both the society and rent seekers to let rent-seeking groups decide freely on releasing their sharing-rule information.

We considered collective rent seeking in which the prize is awarded to one of the players, and the winner shares the prize with the other players in his group. Instead, we can consider collective rent seeking in which the prize is awarded to one of the groups, and the players in the winning group share the prize among themselves. As mentioned in footnote 3, with the sharing rule specification therein, we obtain exactly the same results.

We assumed that both groups consist of the same number of players. What happens if we assume that the groups have different numbers of players? Let us assume that group 1 consists of n_1 players and group 2 consists of n_2 players. First, we can explicitly obtain the results corresponding to Lemmas 1 through 4, some of which involve very long mathematical expressions. Second, part (a) of Proposition 1 holds true, regardless of the values of n_1 and n_2 . Third, because of computational complexity involved, we cannot show explicitly that part (b) of Proposition 1 holds true.²⁴ However, on the basis of our experience of finding the equilibria of the game using different numerical values of the parameters, we believe that it holds true subject to the constraint corresponding to that specified in footnote 13.²⁵ Finally, once part (b) of Proposition 1 holds true, then so do Lemma 5 and Proposition 2. However, it is not generally possible to compare the total effort level in the selected equilibrium with those obtained in the other three subgames that are not specified in the selected equilibrium.

It would be interesting to study the following three-stage game and compare its outcomes with those obtained in this paper. In the first stage, the players in each group jointly choose and commit to their sharing rule. In the second stage, each group decides independently whether it will release to the rival group the information about its sharing rule. Each group then releases the information about its sharing rule if it decided to do so. In the third stage, all the players in both groups choose their effort levels simultaneously and independently. It would also be interesting to examine the groups' decisions on releasing sharing-rule information in a production and conflict model in which each player allocates his resource between production and fighting. We leave these for future research.

Footnotes

1. Nitzan (1994) provides an excellent survey of the literature on rent seeking.
2. Considering contests in which two players compete for a prize by choosing their effort levels simultaneously, Dixit (1987) defines the favorite [the underdog] as the player who has a probability of winning greater [less] than 1/2 at the Nash equilibrium.
3. Baik (1994), Baik and Lee (2001), and Baik et al. (2006) study collective rent seeking in which the prize is awarded to one of the players. Alternatively, we can develop a model in which the prize is awarded to one of the *groups*; the players in the winning group share the prize among themselves; the fractional share of player k is determined by

$$\lambda_k = \theta x_k / X + (1 - \theta) / n,$$

where x_k represents the effort level expended by player k in the winning group, $X = \sum_{k=1}^n x_k$, and the parameter θ is chosen by the players at the beginning of the contest. This sharing rule specification is used in Nitzan (1991a, 1991b), Baik and Shogren (1995), Hausken (1995), Lee (1995), Davis and Reilly (1999), Ueda (2002), Baik and Lee (2007), and Nitzan and Ueda (2007). Note that, in this alternative model, the players in the winning group need to know how much effort each player expended when they share the prize, while this is not the case with the model under consideration. Utilizing the results in Baik et al. (2006), one can see that this alternative model yields exactly the same (main) results.

4. This logit-form contest success function is extensively used in the literature on rent seeking. Examples include Tullock (1980), Appelbaum and Katz (1987), Hillman and Riley (1989), Hirshleifer (1989), Katz, Nitzan, and Rosenberg (1990), Nitzan (1991a, 1991b), Leininger (1993), Baik (1994), Lee (1995), Che and Gale (1997), Hurley and Shogren (1998), Davis and Reilly (1999), Baik and Lee (2001, 2007), Morgan (2003), and Stein and Rapoport (2004).

5. We obtain the same results with the assumption that $\sigma_i > 0$. However, the current assumption makes the analysis simpler.

6. Why do such groups exist? Why are such groups formed? Baik and Lee (2001) provide two reasons. First, depending on the sharing rule, each player in such a group can share with the other members the risk of his failure in winning the prize, or can earn more than the prize when he becomes the winner. Second, the players in such a group can benefit by achieving strategic commitments through their sharing rule.

Examples of such groups include political parties, R&D joint ventures among firms, and coalitions among political parties or interest groups. Some states in the United States provide their colleges and universities with state funds to match federal awards for research and research equipment. This gives another example. We can consider each state as a group in our model, and its colleges and universities as the players in the group which compete for federal research awards. College football conferences are yet another example.

7. Note that, because the players in each group are identical, their decision on the sharing rule is unanimous.

8. Thus, the game is overall a simultaneous-move game between the two groups. Furthermore, because the game has sequential moves, it is a simultaneous-move game with sequential moves. Baik and Lee (2007) study a general model of the simultaneous-move game with sequential moves.

9. It is straightforward to see that π_{ik} in Equation (1) is strictly concave in x_{ik} , and thus the second-order condition for maximizing Equation (1) is satisfied. Certainly, the second-order condition is satisfied for every maximization problem in the paper, although we do not state so explicitly in each case for concise exposition.

10. In the Appendix, we set up and analyze a general model of the game between two parties in which each party has two sequential moves and both parties' first moves are not public information.

11. In this way of solving the (R, NR) subgame, group 1 is treated as the strategical leader that chooses its sharing rule before group 2 does. This may involve assuming that group 2 is not committed to a sharing rule until group 1 announces its sharing rule publicly.

12. Using Equation (13), we see that the players in group 1 expend zero effort when $v_2(\sigma_2 - 1/n) \geq \gamma v_1 \sigma_1$; the players in group 2 expend zero effort when $\gamma v_1(\sigma_1 - 1/n) \geq v_2 \sigma_2$. Such "monopolization" in collective rent seeking was first studied by Ueda (2002). Because monopolization does not occur in the subgame-perfect equilibrium of the bilateral-release subgame, we omit a complete description of the symmetric Nash equilibrium of the second stage for concise exposition.

13. In Lemma 4, both x_1^{BR} and x_2^{BR} are positive only when $n/(n+1) < \alpha\gamma < (n+1)/n$. Therefore, Proposition 1 is valid if and only if $n/(n+1) < \alpha\gamma < (n+1)/n$.

14. Note that π_i^{BR} is always smaller than π_i^{NR} , smaller than π_i^{1R} , and smaller than π_i^{2R} , for $i = 1, 2$.

15. See, for example, Fershtman and Judd (1987) and Sklivas (1987). Yildirim (2005) studies the following two-player contest. In period 0, each player decides whether he will release the information about his period-1 effort level to the rival. Then the players announce their decisions simultaneously. In period 1, knowing which player will or will not release the information, the players simultaneously choose their effort levels. Then each player releases the information about his effort level if he decided to do so in period 0. In period 2, the players simultaneously choose their effort levels again – that is, each player adds zero or positive effort to his period-1 effort level. Interestingly, Yildirim finds that both players decide to release the information in equilibrium.

16. Indeed, if $\alpha\gamma > 1$, then each player expends a smaller effort level in the equilibrium involving (NR, R) than in the equilibrium involving (R, NR) ; if $\alpha\gamma < 1$, then the opposite holds true. In terms of symbols, if $\alpha\gamma > 1$, then we have $x_i^{2R} < x_i^{1R}$ for $i = 1, 2$; if $\alpha\gamma < 1$, then we have $x_i^{1R} < x_i^{2R}$ for $i = 1, 2$.

17. Baik and Shogren (1992) and Leininger (1993) study contests with two asymmetric players in which the players first announce publicly when they will exert their effort, and then based on this timing, they choose their effort levels. They find that the underdog always exerts effort before the favorite does.

18. The favorite and underdog choose the same winner's fractional share if, and only if, $n = 2$.

19. If $\alpha\gamma = 1$, then the groups choose the same winner's fractional share in equilibrium, which is equal to $(n + 1)/2n$. Note that the equilibrium winner's fractional share depends only on the size n of the groups; it is greater than a half, but less than unity. If $\alpha\gamma = 1$, then the players in both groups have the same probability of winning in equilibrium.

20. More precisely, $X_1^{2R} + X_2^{2R} < X_1^{BR} + X_2^{BR}$ in the first part holds when $\alpha \geq 0.375$, and $X_1^{1R} + X_2^{1R} < X_1^{BR} + X_2^{BR}$ in the second part holds when $1/\alpha \geq 0.375$. If $\alpha\gamma = 1$, then we have $x_i^{NR} = x_i^{1R} = x_i^{2R} < x_i^{BR}$ for $i = 1, 2$. This implies that the total effort level is smaller in the equilibria than in the (R, R) subgame.

21. In the literature on rent seeking, examining the extent of rent dissipation is one of the main issues because the opportunity costs of resources expended on rent-seeking activities are viewed as social costs. Many papers show that less than complete dissipation of the contested rent occurs. Examples include Tullock (1980), Hillman and Riley (1989), Baik and Lee (2001), and Baik (2004).

22. Note that the (NR, NR) subgame is the case where sharing rules are private information and the (R, R) subgame is the case where sharing rules are public information.

23. We thank one of the referees for pointing out this possible commitment problem. In general, a commitment problem refers to a situation in which players cannot "achieve their goals" because of their inability to (credibly) commit to their actions.

24. It is computationally intractable to show explicitly that π_1^{2R} is greater than or equal to π_1^{BR} , and that π_2^{1R} is greater than or equal to π_2^{BR} .

25. We investigated using more than ten different sets of numerical values, and found that it held true for all the numerical cases considered.

Appendix

A1. The game between two parties in which both parties' first moves are not public information

Consider a game between two parties, 1 and 2, in which each party has two sequential moves. The first move of one party is observed by all the players in both parties before the second moves of the parties are chosen. However, the first move of the other party is observed only by the players in that party; it is *hidden* from the players in the rival party. The second moves of the parties are chosen *simultaneously*. For expositional convenience, the player or group of players in party i , for $i = 1, 2$, who chooses the first move is called leader i , and that choosing the second move is called follower i . We allow leader i and follower i to be the same player or group of players.

We formally consider the following game. First, leaders 1 and 2 choose actions a_1 and a_2 from A_1 and A_2 , respectively, where A_i denotes the set of all actions available to leader i . Next, follower 1 observes the action a_1 chosen by leader 1, but cannot observe the action a_2 chosen by leader 2. Follower 2, however, observes both a_1 and a_2 . Finally, followers 1 and 2 simultaneously choose actions b_1 and b_2 from B_1 and B_2 , respectively, where B_i is follower i 's set of actions. Let u_i represent the (expected) payoff for leader i and v_i that for follower i . The payoff function for leader i and that for follower i are given by $u_i = u_i(a_i, b_i, b_j)$ and $v_i = v_i(a_i, b_i, b_j)$, respectively. (Throughout the paper, when we use i and j at the same time, we mean that $i \neq j$.) Note that a_j is absent in these functions. This implies that the payoffs to the players in each party do not depend *directly* on the first move of the other party. We assume that all of the above is common knowledge among the leaders and followers.

The right way to look at this game is that the two parties play the following two-stage game. In the first stage, leader 1 chooses her action. In the second stage, after observing leader 1's action, follower 1 and *party 2* play a simultaneous-move game: Follower 1 chooses his action without observing party 2's sequential actions, and party 2 – specifically, leader 2 and follower 2 – chooses its two sequential actions without observing follower 1's action. In this way of solving the game, leader 1 is treated as the strategical leader who moves even before leader 2 moves, whatever the chronological timing of their decisions may be in the original game.

A2. Equilibrium actions

To solve the two-stage game, we take the following three steps. First, we analyze the subgames which start at the second stage of the game. In each of the subgames, follower 1 and party 2 play a simultaneous-move game, knowing the action of leader 1 which gave rise to the subgame. To solve the subgames, we need to find a triple vector, $(b_1^*(a_1), a_2^*(a_1), b_2^*(a_1))$, which satisfies the following two requirements. First, for any action a_1 of leader 1, follower 1's action $b_1^*(a_1)$ is optimal given the action $b_2^*(a_1)$ of follower 2; follower 2's action $b_2^*(a_1)$ is optimal given the action $a_2^*(a_1)$ of leader 2 and given the action $b_1^*(a_1)$ of follower 1. That is, for any action a_1 of leader 1, follower 1's action $b_1^*(a_1)$ is a best response to follower 2's action $b_2^*(a_1)$; follower 2's action $b_2^*(a_1)$ is a best response to leader 2's action $a_2^*(a_1)$ and follower 1's action $b_1^*(a_1)$. Second, for any action a_1 of leader 1, leader 2's action $a_2^*(a_1)$ is optimal given the action $b_1^*(a_1)$ of follower 1 and given the subsequent behavior of follower 2. Note that such a triple vector permits the interpretation that for any action a_1 of leader 1, $b_1^*(a_1)$ is a best response to $(a_2^*(a_1), b_2^*(a_1))$ and vice versa.

Next, we analyze the first stage in which leader 1 chooses her action. We need to find leader 1's action a_1^* which is optimal given the strategy $a_2^*(a_1)$ of leader 2, the strategy $b_1^*(a_1)$ of follower 1, and $b_2^*(a_1)$ of follower 2.

Finally, we obtain the equilibrium actions of the two-stage game – and thus those of the original game – using the findings in the previous two steps. The equilibrium actions are $(a_1^*, b_1^*(a_1^*), a_2^*(a_1^*), b_2^*(a_1^*))$.

A2.1. Solving the subgames starting at the second stage

In the second stage, follower 1 and party 2 play a simultaneous-move game, knowing the action a_1 chosen by leader 1 in the first stage. To obtain follower 1's reaction function, consider his maximization problem. Knowing the action a_1 , follower 1 seeks to maximize his payoff over his action b_1 , taking follower 2's action b_2 as given:

$$\max_{b_1 \in B_1} v_1(a_1, b_1, b_2). \quad (\text{A1})$$

We assume that for each a_1 in A_1 and b_2 in B_2 , maximization problem (A1) has a unique interior solution, which is denoted by

$$b_1^{BR}(a_1, b_2). \quad (\text{A2})$$

This is follower 1's best response to follower 2's action b_2 , given the action a_1 of leader 1. Note that, because the payoff to follower 1 does not depend directly on leader 2's action a_2 , it is also follower 1's best response to party 2's pair of actions (a_2, b_2) , given the action a_1 of leader 1. Now that follower 1's reaction function shows his best response to every possible action that follower 2 might choose, it comes from follower 1's best response (A2). We denote it by

$$b_1 = b_1^{BR}(b_2; a_1). \quad (\text{A3})$$

To obtain party 2's reaction functions – specifically, the reaction function for leader 2 and that for follower 2 – we need to consider two separate but related maximization problems. Working backward, consider first follower 2's maximization problem. Knowing the action a_2

chosen by leader 2, follower 2 seeks to maximize his payoff over his action b_2 , taking follower 1's action b_1 as given:

$$\max_{b_2 \in B_2} v_2(a_2, b_1, b_2). \quad (\text{A4})$$

We assume that for each a_2 in A_2 and b_1 in B_1 , maximization problem (A4) has a unique interior solution, which is denoted by

$$b_2^{BR}(a_2, b_1). \quad (\text{A5})$$

This is follower 2's best response to leader 2's action a_2 and follower 1's action b_1 .

Next, consider leader 2's maximization problem. Leader 2 seeks to maximize her payoff over her action a_2 , taking follower 1's action b_1 as given:

$$\max_{a_2 \in A_2} u_2(a_2, b_1, b_2). \quad (\text{A6})$$

Because leader 2 can solve follower 2's maximization problem (A4) as well as follower 2 can, leader 2 has perfect foresight about follower 2's best response to each action a_2 that she might take – that is, she knows in advance $b_2^{BR}(a_2, b_1)$ for each action a_2 . Thus, leader 2's maximization problem (A6) amounts to

$$\max_{a_2 \in A_2} u_2(a_2, b_1, b_2^{BR}(a_2, b_1)). \quad (\text{A7})$$

We assume that for each b_1 in B_1 , maximization problem (A7) has a unique interior solution, which is denoted by

$$a_2^{BR}(b_1). \quad (\text{A8})$$

This is leader 2's best response to follower 1's action b_1 .

Follower 2's reaction function shows his best response to every possible pair of actions that leader 2 and follower 1 might choose. Thus, it comes from follower 2's best response (A5):

$b_2 = b_2^{BR}(a_2, b_1)$. Similarly, leader 2's reaction function shows her best response to every possible action that follower 1 might choose, and thus it comes from leader 2's best response (A8): $a_2 = a_2^{BR}(b_1)$. Therefore, the reaction functions for party 2 are

$$a_2 = a_2^{BR}(b_1) \tag{A9}$$

and

$$b_2 = b_2^{BR}(a_2, b_1). \tag{A10}$$

Now we obtain the triple vector, $(b_1^*(a_1), a_2^*(a_1), b_2^*(a_1))$, using the three reaction functions, (A3), (A9) and (A10). Specifically, we obtain it by solving the system of three simultaneous equations which consists of (A3), (A9) and (A10).

A2.2. *The first stage and the equilibrium actions of the game*

Consider the first stage in which leader 1 chooses her action. Leader 1 seeks to maximize her payoff over her action a_1 :

$$\max_{a_1 \in A_1} u_1(a_1, b_1, b_2). \tag{A11}$$

Because leader 1 can solve the subgames starting at the second stage, she knows in advance $(b_1^*(a_1), a_2^*(a_1), b_2^*(a_1))$ for each action a_1 that she might take. Thus, leader 1's maximization problem (A11) amounts to

$$\max_{a_1 \in A_1} u_1(a_1, b_1^*(a_1), b_2^*(a_1)). \tag{A12}$$

We assume that maximization problem (A12) has a unique interior solution, which is denoted by a_1^* .

Finally, substituting a_1^* into the triple vector, $(b_1^*(a_1), a_2^*(a_1), b_2^*(a_1))$, obtained in Section A2.1, we obtain the equilibrium actions of the game, $(a_1^*, b_1^*(a_1^*), a_2^*(a_1^*), b_2^*(a_1^*))$.

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TABLE 1

The Strategic Interaction Between the Groups in the First Stage

		Group 2	
		<i>NR</i>	<i>R</i>
Group 1	<i>NR</i>	π_1^{NR}, π_2^{NR}	π_1^{2R}, π_2^{2R}
	<i>R</i>	π_1^{1R}, π_2^{1R}	π_1^{BR}, π_2^{BR}

TABLE 2

The Outcomes of the Contest in the Selected Equilibrium

	$\alpha\gamma > 1$	$\alpha\gamma < 1$
Favorite	Group 1	Group 2
First-Stage Combination	(NR, R)	(R, NR)
Strategical Leader	Group 2	Group 1
Winner's Fractional Shares	$0.5 < \sigma_1^{2R} \leq \sigma_2^{2R} < 1$	$0.5 < \sigma_2^{1R} \leq \sigma_1^{1R} < 1$
Probabilities of Winning	$p_2^{2R} < p_1^{2R}$	$p_1^{1R} < p_2^{1R}$