1.  (1) To find the monopolist’s profit-maximizing output level, apply the three conditions. Find a level of output, \( y^o \), where (i) \( MR(y^o) = MC(y^o) \), and (ii) the marginal cost curve cuts its marginal revenue curve from below. (iii) If \( p(y^o) \geq AVC(y^o) \), then the profit-maximizing output level is \( y^o \); otherwise, it is zero. (You may use other equivalent conditions for (iii).)

The profit-maximizing output level is: \( y^* = (a - c)/2b \).

(2) The monopolist's profit-maximizing price is: \( p(y^*) = a - by^* = (a + c)/2 \).

(3) \( \epsilon(y^*) = \{p(y^*)/y^*\}(dy/dp) = -(a + c)/(a - c) \)

(4) The deadweight loss due to the monopoly is: \( DWL = (a - c)^2/8b \).

2.  (1) The monopolist maximizes its profits, \( \pi(q) \), over its output level, where \( \pi(q) = p(q)q - c(q) \).

The first-order (necessary) condition for maximizing \( \pi(q) \) is

\[ p(q) + \{dp(q)/dq\}q = c'(q) \]

This implies that at its profit-maximizing output level, \( q^m \), we must have

\[ p(q^m) + \{dp(q^m)/dq\}q^m = c'(q^m) \]

Rearranging this, we get

\[ \{p(q^m) - c'(q^m)\}/p^m = -\{dp(q^m)/dq\}\{q^m/p^m\} \]

where \( p^m \equiv p(q^m) \).

Now let \( \epsilon(q) = \{dq/dp\}\{p(q)/q\} \) be the elasticity of demand. Then we obtain

\[ \{p^m - c'(q^m)\}/p^m = -1/\epsilon(q^m) \]

(A1)

(Note that \( \epsilon(q) \leq 0 \), and that \( \epsilon(q^m) = \epsilon(p^m) \).

(2) If \( c'(q) > 0 \) at all \( q \geq 0 \), then we have

\[ \{p^m - c'(q^m)\}/p^m < [p^m - 0]/p^m = 1 \]

It follows from expression (A1) that \( \epsilon(q^m) < -1 \) or, equivalently, \( |\epsilon(q^m)| > 1 \). This says that demand is elastic at the monopolist’s optimal quantity.
3. The inverse elasticity rule is
\[ p_m = MC(y_m)/(1 + 1/\varepsilon(y_m)). \]

When the monopoly is subject to an ad valorem tax of \( t \), this becomes
\[ p_{m \text{ after-tax}} = MC(y_{m \text{ after-tax}})/(1 - t)(1 + 1/\varepsilon(y_{m \text{ after-tax}})). \]

(1) With linear demand, the elasticity of demand \( \varepsilon(y) \) falls as output falls. The ad valorem tax reduces the monopolist's optimal level of output:
\[ y_{m \text{ after-tax}} < y_{m \text{ pretax}}. \]
Hence, \( p_{m \text{ after-tax}} = MC/(1 - t)(1 + 1/\varepsilon(y_{m \text{ after-tax}})) \) is less than \( MC/(1 - t)(1 + 1/\varepsilon(y_{m \text{ pretax}})) \).

Because the latter expression is equal to \( p_{m \text{ pretax}}/(1 - t) \), we have
\[ p_{m \text{ after-tax}} < p_{m \text{ pretax}}/(1 - t). \]

(2) With constant elasticity demand, we have
\[ p_{m \text{ after-tax}} = MC/(1 - t)(1 + 1/\varepsilon) = p_{m \text{ pretax}}/(1 - t). \]
Explain.

(3) Assume that the monopoly faces a constant price elasticity demand curve.

If the monopoly operates on a negatively sloped portion of its marginal cost curve, then \( p_{m \text{ after-tax}} = MC(y_{m \text{ after-tax}})/(1 - t)(1 + 1/\varepsilon) \) is greater than \( MC(y_{m \text{ pretax}})/(1 - t)(1 + 1/\varepsilon) \).

(Note that \( y_{m \text{ after-tax}} < y_{m \text{ pretax}} \).)

Because \( MC(y_{m \text{ pretax}})/(1 - t)(1 + 1/\varepsilon) \) is equal to \( p_{m \text{ pretax}}/(1 - t) \), we have
\[ p_{m \text{ after-tax}} > p_{m \text{ pretax}}/(1 - t). \]