1. (1) It means that the two types of consumers pick different price-quantity packages; furthermore, each consumer chooses the price-quantity package meant for him or her.

(2) To maximize its profits, the monopolist offers an amount of 0 at a price of 0 and an amount of 100 at a price of 5,000. The first price-quantity package is targeted toward type 1, and the second package toward type 2. Note that offering the two price-quantity packages is equivalent to offering only one price-quantity combination, an amount of 100 at a price of 5,000.

To prove this answer, we first compute the monopolist's profits when it offers an amount $x_1$ at price $(100 - x_1)/2$ and an amount of 100 at price $(100 - x_1)^2/2$, where $x_1 \leq 50$.

(Note that $(100 - x_1)/2 + (100 - x_1)^2/2 = 50(100 - x_1))

In this case, because type 1 finds it optimal to choose $x_1$ and pay price $(100 - x_1)/2$, and type 2 finds it optimal to choose 100 and pay price $50(100 - x_1)$, the monopolist's profits are $\pi = 500,000 - 50(x_1)^2$.

The monopolist's profits increases as $x_1$ decreases: $d\pi/dx_1 < 0$.

This means that the monopolist's profits are less than 500,000 for any $x_1 > 0$. Hence, the monopolist earns more profits by offering the single price-quantity combination, an amount of 100 at a price of 5,000.

(This holds true unless the consumers of type 1 outnumber "sufficiently" those of type 2.)

2. (1) In equilibrium, each firm charges a price of zero.

(2) Using the equations, $p_1 + cx = p_2 + cy$ and $a + x + y + b = L$, we obtain

$$x = \frac{L - a - b + (p_2 - p_1)/c_1}{2}$$

and

$$y = \frac{L - a - b + (p_1 - p_2)/c_1}{2}.$$
The firms' profits are then
\[ \pi_1 = p_1(a + x) = (L + a - b)p_1/2 + (p_1p_2 - p_1^2)/2c \]
and
\[ \pi_2 = p_2(b + y) = (L - a + b)p_2/2 + (p_1p_2 - p_2^2)/2c. \]  
(A1)

Next, firm i's reaction function (or its best response function), for \( i = 1, 2 \), is derived from the first-order condition for maximizing \( \pi_i \) over its price \( p_i \).

Given firm 2's price \( p_2 \), the first-order condition for maximizing \( \pi_1 \) over \( p_1 \) is
\[ \partial \pi_1 / \partial p_1 = (L + a - b)/2 + p_2/2c - p_1/c = 0. \]  
(A2)

(Check whether the second-order condition for maximizing \( \pi_1 \) is satisfied.)

Similarly, given firm 1's price \( p_1 \), the first-order condition for maximizing \( \pi_2 \) over \( p_2 \) is
\[ \partial \pi_2 / \partial p_2 = (L - a + b)/2 + p_1/2c - p_2/c = 0. \]  
(A3)

Using (A2) and (A3), we obtain the firms' reaction functions:
\[ p_1 = c(L + a - b)/2 + p_2/2 \]
and
\[ p_2 = c(L + a - b)/2 + p_1/2. \]

Now we obtain the Nash equilibrium, \( (p_1^*, p_2^*) \), by solving the system of two simultaneous equations which come from the firms' reaction functions:
\[ p_1^* = c\{L + (a - b)/3\} \]
and
\[ p_2^* = c\{L - (a - b)/3\}. \]  
(A4)

Using (A1) and (A4), we obtain the firms' profits at the Nash equilibrium:
\[ \pi_1^* = p_1^*(a + x^*) = c(3L + a - b)^2/18 \]
and
\[ \pi_2^* = p_2^*(b + y^*) = c(3L - a + b)^2/18. \]

The pair \( (p_1^*, p_2^*) \) of prices constitutes a pure-strategy Nash equilibrium if and only if firm i has no incentive to deviate from its price \( p_i^* \), given the other firm's price \( p_j^* \), for \( i, j = 1, 2 \) with \( i \neq j \).

Here it suffices to derive the conditions on the parameters under which it does not pay for firm i to attempt taking over the whole market, given the other firm's price \( p_j^* \).
Suppose that, given firm 2's price $p^*_2$, firm 1 charges $p^*_1$, where $p^*_1 = p^*_2 - c(L - a - b) > 0$.

Then, under our "tie-breaking rule," firm 1 serves the entire market, so that its profits are $\pi_1^o = p^*_1 L = cL(2a + 4b)/3$.

Similarly, suppose that, given firm 1's price $p^*_1$, firm 2 charges $p^*_2$, where $p^*_2 = p^*_1 - c(L - a - b) > 0$.

Then, under our "tie-breaking rule," firm 2 serves the entire market, so that its profits are $\pi_2^o = p^*_2 L = cL(4a + 2b)/3$.

Now the pair $(p^*_1, p^*_2)$ of prices constitutes a pure-strategy Nash equilibrium if and only if $\pi_1^* \geq \pi_1^o$ and $\pi_2^* \geq \pi_2^o$,

which lead to $(L + (a - b)/3)^2 \geq 4L(a + 2b)/3$ and $(L - (b - a)/3)^2 \geq 4L(b + 2a)/3$, respectively.

(4) No, the equilibrium is not efficient.
Assume that $a > b$. Then we have $p^*_1 > p^*_2$ and $x^* < y^*$.
In this case, the equilibrium is inefficient in that a consumer slightly to the right of $E$ would incur a shorter walk by patronizing firm 1, but still chooses firm 2 because of firm 1's power to set higher prices.

3. (1) In equilibrium, each firm charges a price of zero.

(2)

Since the consumer located at point $x$ is indifferent between buying from the two firms, the following equation must be satisfied:

$p_1 + c(x - a)^2 = p_2 + c(b - x)^2$.

Solving this equation for $x$ yields

$x = (a + b)/2 + (p_2 - p_1)/2c(b - a)$.

The firms' profits are then

$\pi_1 = p_1 x = (a + b)p_1/2 + (p_2 - p_1)p_1/2c(b - a)$

and

$\pi_2 = p_2(L - x) = Lp_2 - (a + b)p_2/2 + (p_1 - p_2)p_2/2c(b - a)$.
Next, firm $i$'s reaction function (or its best response function), for $i = 1, 2$, is derived from the first-order condition for maximizing $\pi_i$ over its price $p_i$.

Given firm 2's price $p_2$, the first-order condition for maximizing $\pi_1$ over $p_1$ is
\[
\frac{\partial \pi_1}{\partial p_1} = \frac{(a + b)}{2} + \frac{p_2}{2c(b - a)} - \frac{p_1}{c(b - a)} = 0. \tag{B2}
\]

(Check whether the second-order condition for maximizing $\pi_1$ is satisfied.)

Similarly, given firm 1's price $p_1$, the first-order condition for maximizing $\pi_2$ over $p_2$ is
\[
\frac{\partial \pi_2}{\partial p_2} = L - \frac{(a + b)}{2} + \frac{p_1}{2c(b - a)} - \frac{p_2}{c(b - a)} = 0. \tag{B3}
\]

Using (B2) and (B3), we obtain the firms' reaction functions:
\[
p_1 = c(a + b)(b - a)/2 + p_2/2
\]
and
\[
p_2 = c(2L - a - b)(b - a)/2 + p_1/2.
\]

Now we obtain the Nash equilibrium, $(p_1^*, p_2^*)$, by solving the system of two simultaneous equations which come from the firms' reaction functions:
\[
p_1^* = c(b - a)(2L + a + b)/3
\]
and
\[
p_2^* = c(b - a)(4L - a - b)/3. \tag{B4}
\]

Using (B1) and (B4), we obtain the firms' profits at the Nash equilibrium:
\[
\pi_1^* = p_1^*x^* = \frac{c(b - a)(2L + a + b)^2}{18}
\]
and
\[
\pi_2^* = p_2^*(L - x^*) = \frac{c(b - a)(4L - a - b)^2}{18}.
\]